Alberta Provincial Achievement Testing

Assessment Highlights 2013–2014

GRADE 9

Mathematics

This document contains assessment highlights from the 2014 Grade 9 Mathematics Achievement Test. The examination statistics included in this document represent all writers, both French and English. To obtain English-only or French-only statistics that apply to your school, please refer to your detailed reports, which are available on the extranet.

Assessment Highlights provides information about the overall test, test blueprints, and student performance on the achievement test that was administered in 2014. Also provided is commentary on student performance at the acceptable standard and the standard of excellence on selected items from the 2014 Mathematics Achievement Test. This information is intended for teachers and is best used in conjunction with multi-year and detailed school reports that are available in schools via the extranet. *Assessment Highlights* reports for all achievement test subjects and grades will be posted on the Alberta Education website every year in the fall.

All released achievement tests, including test blueprints, answer keys with the item difficulty, reporting category, test section, and item description for each test item, are located at education.alberta.ca/admin/testing/achievement/answerkeys.aspx.

These materials, along with the program of studies and subject bulletins, provide information that can be used to inform instructional practice.

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The Alberta Education Internet address is education.alberta.ca.

This document was written primarily for:

Students	
Teachers	✓ of Grade 9 Mathematics
Administrators	✓
Parents	
General Audience	
Others	

Contents

The 2014 Grade 9 Mathematics Achievement Test

This report provides teachers, school administrators, and the public with an overview of the performance of those students who wrote the 2014 Grade 9 Mathematics Achievement Test. It complements the detailed school and jurisdiction reports.

How Many Students Wrote the Test?

A total of 38 646 students wrote the 2014 Grade 9 Mathematics Achievement Test. The English form of the test was written by 35 881 students, and the French form of the test was written by 2 765 students.

What Was the Test Like?

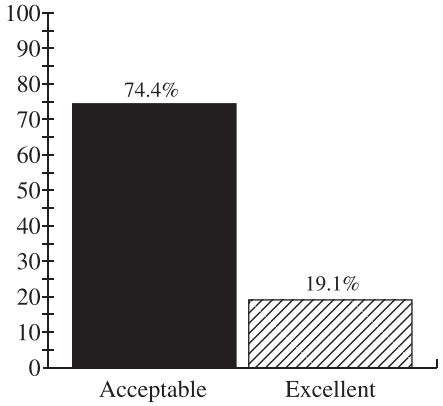
The 2014 Grade 9 Mathematics Achievement Test consisted of 40 multiple-choice and 10 numerical-response questions based on four strands: Number, Patterns and Relations, Shape and Space, and Statistics and Probability. In keeping with the intent of the 2007 Program of Studies, the questions on the test required students to apply their understanding of one or more mathematical concepts from within and/or across the four strands. As they solved the mathematical problems, students were expected to use the interrelated mathematical processes of Communication, Connections, Mental Mathematics and Estimation, Problem Solving, Reasoning, and Visualization. A detailed explanation of these mathematical processes is in the Alberta K–9 Mathematics Program of Studies.

How Well Did Students Do?

The percentages of students meeting the acceptable standard and the standard of excellence in 2014 are shown in the graph below. Out of a total score of 50 on the test, the provincial average was 29.96/50 (59.9%). The results presented in this report are based on scores achieved by all students who wrote the test, including those in French Immersion and Francophone programs. Detailed provincial assessment results are provided in school and jurisdiction reports.

Grade 9—2014 Mathematics Achievement Test		
	Acceptable (%)	Excellent (%)
2014	74.4	19.1

Percentage of Students Meeting the Acceptable Standard & Standard of Excellence (%)



2014 Achievement Standards: The percentage of students in the province who met the acceptable standard on the 2014 Grade 9 Mathematics Achievement Test (based on those who wrote)

2014 Achievement Standards: The percentage of students in the province who met the standard of excellence on the 2014 Grade 9 Mathematics Achievement Test (based on those who wrote)

2014 Test Blueprint and Student Achievement

In 2014, 74.4% of students who wrote the test achieved the acceptable standard on the Grade 9 Mathematics Achievement Test, and 19.1% of students who wrote achieved the standard of excellence.

Out of a total score of 50 on the test, the provincial average was 29.96/50 (59.9%). The blueprint below shows how the questions on the test were classified and includes the average raw score in each category for all Grade 9 students who wrote this test.

Strand	Level of Complexity*			Provincial Student Achievement (Raw Score and Percentage)
	Low	Moderate	High	
Number	**5**	**7**	**2**	**9.2/15 (61.3%)**
Patterns and Relations	**6**	**11**	**3**	**12.2/20 (61.0%)**
Shape and Space	**6**	**3**	**2**	**5.9/11 (53.6%)**
Statistics and Probability	**3**	**1**	**0**	**2.4/4 (60.0%)**
Provincial Student Achievement (Average Raw Score and Percentage)	**12.2/20 (61.0%)**	**13.3/23 (57.8%)**	**4.3/7 (61.4%)**	**Total Test Raw Score 29.96/50 (59.9%)**

*Each question is categorized according to its level of complexity (low, moderate, or high). Descriptions of the levels of complexity are in the *2014–2015 Mathematics 9 Subject Bulletin*.

2014 Grade 9 Mathematics Achievement Test Design Commentary

The 2014 Mathematics Provincial Achievement Test for Grade 9 was based on the 2007 Alberta K–9 Mathematics Program of Studies that was implemented in the 2011–2012 school year. The test blueprint provides information about new test design features (i.e., complexity) and modified test design features (i.e., item format and strand). Items now are selected not only in terms of the knowledge and skills that they assess, but also in terms of their complexity in the form of content and cognition. The introduction of item complexity will provide more information about the depth to which students have mastered particular learning outcomes, as well as provide one more control in the selection of test items to better ensure that tests are equivalent from year to year. Please refer to the *2014–2015 Mathematics 9 Subject Bulletin* for more-detailed information about item complexity.

The selection of test items within each of the four strands is now based on two primary factors: item difficulty and item complexity.

> **Item difficulty** refers to the percentage of students who actually chose the correct answer. Items for which the correct answer is selected by more than 70% of the students are generally considered easy. Items for which the correct answer is selected by 50–70% of the students are about average in difficulty. Items for which the correct answer is selected by less than 50% of the students are regarded as challenging.
>
> **Item complexity** refers to the cognitive and content demands associated with an item. The rationale for classifying items by their level of complexity is to focus on the expectations underlying the item and not the ability of the student. The cognitive demands that an item makes on a student (i.e., what an item requires the student to recall, understand, analyze, and do) are made with the assumption that the student is familiar with the basic concepts of the task.

The categories—low complexity, moderate complexity, and high complexity—form an ordered description of the demands an item may make on a student. For example, low-complexity items may require a student to solve a one-step problem. Moderate-complexity items may require multiple steps. High-complexity items go even further and require a student to analyze and synthesize information. It is therefore important to consider both the content being assessed by an item and the item complexity when making inferences about student performance on any one outcome. Although there is a logical and predictable relationship between item difficulty and item complexity (i.e., items that are of high complexity tend to be more challenging), there are exceptions.

The following eight items have been released to illustrate significant performance differences between two groups of students: (1) those students who achieved the standard of excellence as opposed to those who achieved the acceptable standard, and (2) those students who achieved the acceptable standard as opposed to those who were below the acceptable standard. The purpose of these comparisons is to provide additional information that may be used for instructional purposes.

Sample Questions from the 2014 Grade 9 Mathematics Achievement Test

The following four items, from all four strands, illustrate significant performance differences between students who achieved the standard of excellence and those who achieved the acceptable standard.

Item #	Strand	Specific Outcome	Item Complexity	Item Description
19	PR	3	Moderate	Determine the solution to a given problem by relating two linear equations to each other to solve for an unknown variable.

	% of Student Responses (*Correct)			
	A	**B***	**C**	**D**
Performance of Students Achieving the Standard of Excellence	10.4	83.6	5.6	0.4
Performance of Students Achieving the Acceptable Standard	38.7	48.4	10.6	2.2

The amount of money, A, Hanna receives selling bracelets, b, at a local market is represented by the relation $A = 5b$. Her expenses, E, for making the bracelets are represented by the relation $E = 20 + b$.

19. What is the minimum number of bracelets that Hanna needs to sell to pay for her expenses?

A. 4 bracelets

B. 5 bracelets

C. 6 bracelets

D. 7 bracelets

Commentary:

Over four-fifths of the students who achieved the standard of excellence were successful on this item, while less than half of the students who achieved the acceptable standard were successful. Two-step items such as this, where students are to translate a verbal description into an inequality and then solve the inequality, are relatively straightforward for students who achieved the standard of excellence, but are difficult for students who achieved the acceptable standard.

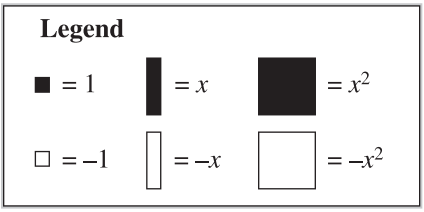
Of the students who achieved the standard of excellence but answered the item incorrectly, almost all chose either option A or option C. These responses suggest that students were able to set up an inequality with $5b$ on the left and $b + 20$ on the right. In choosing incorrect options, twice as many of these students set up $5b < b + 20$, giving option A, as set up $5b > b + 20$, giving option C.

Likewise, of the students who achieved the acceptable standard but answered the item incorrectly, almost all chose either option A or option C. These responses suggest that students were able to set up an inequality with $5b$ on the left and $b + 20$ on the right. In choosing incorrect options, four times as many of these students set up $5b < b + 20$, giving option A, as set up $5b > b + 20$, giving option C.

It appeared that very few students, at either level, were unable to translate the problem context into a reasonable inequality.

Item #	Strand	Specific Outcome	Item Complexity	Item Description
24	PR	7	Moderate	Given an incomplete model of a multiplication of a polynomial by a monomial, determine the value of the coefficient of the product.

	% of Student Responses (*Correct)			
	A	**B**	**C**	**D***
Performance of Students Achieving the Standard of Excellence	32.6	1.2	8.2	58.0
Performance of Students Achieving the Acceptable Standard	46.3	9.4	17.3	26.8



The diagram below shows an incomplete model of the multiplication of two polynomials.

24. What is the coefficient on the x-term in the product?

A. –12

B. 12

C. –6

D. 6

Commentary:

Over half of the students who achieved the standard of excellence were successful on this item, while barely over a quarter of the students who achieved the acceptable standard were successful. Two-step items such as this, where students have to translate pictorial representations of polynomials into symbolic representations and then carry out an operation on the symbolic representations to obtain a final answer, are difficult for students who achieved the standard of excellence, but are exceptionally difficult for students who achieved the acceptable standard.

Of the students who achieved the standard of excellence but answered the item incorrectly, over three-quarters chose option A as their response. This suggests that these students may have thought that, in order to obtain a term in x, both elements of the product must contain x. This illustrates a common misunderstanding between elements contained in a product and the product itself. Very few of these students made sign errors when multiplying, choosing options B or C.

Likewise, of the students who achieved the acceptable standard but answered the item incorrectly, nearly two-thirds chose option A as their response. One-third of these students who achieved the acceptable standard made sign errors when multiplying, choosing options B or C.

Item #	Strand	Specific Outcome	Item Complexity	Item Description
37	N	2	Moderate	Apply the exponent laws to simplify a given expression involving exponents and identify the resulting exponents.

	% of Student Responses (*Correct)			
	A	B	C*	D
Performance of Students Achieving the Standard of Excellence	6.0	2.4	78.3	13.2
Performance of Students Achieving the Acceptable Standard	20.5	15.7	43.9	19.4

When simplified, the expression $\left[\left(a^2b\right)\left(a^3b^2\right)\right]^3$ can be written in the form a^mb^n.

37. Which of the following rows correctly identifies the values of m and n?

Row	*m*	*n*
A.	8	6
B.	9	5
C.	15	9
D.	18	6

Commentary:

Approximately three-quarters of the students who achieved the standard of excellence were successful on this item, while less than half of the students who achieved the acceptable standard were successful. Multi-step procedural items such as this, where students have to use first the product law of exponents and then the power law on the result, are relatively straightforward for students who achieved the standard of excellence, but are quite difficult for students who achieved the acceptable standard.

Of the students who achieved the standard of excellence but answered the item incorrectly, two-thirds chose option D, indicating they either used $(a^2 \times a^3)^3 = a^{(2 \times 3 \times 3)} = a^{18}$ or took b as equal to b^0 in the first bracket. Other errors in this group of students are less common.

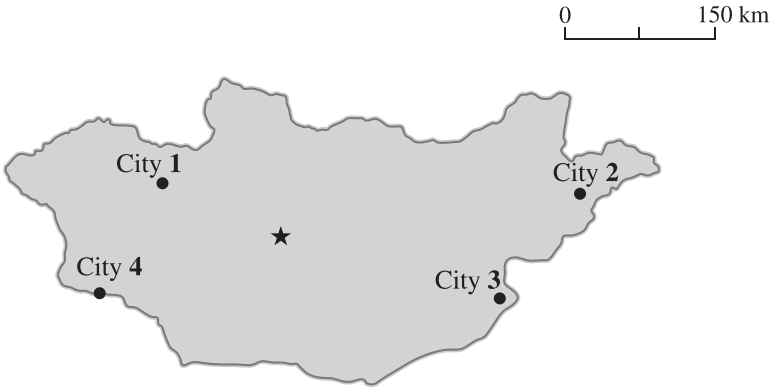
Likewise, of the students who achieved the acceptable standard but answered the item incorrectly, just over one-third chose option D, just over one-third chose option A, and just over one-quarter chose option B. The error of taking b as equal to b^0 in the first bracket can lead to both options A and D, while the error $(a^2 \times a^3)^3 = a^{(2 \times 3 \times 3)} = a^{18}$ leads to

option D, and the error $(a^2 \times a^3)^3 = a^{(2+3+3)} = a^8$ leads to option A. The route to option B is less clear, so the number of these students choosing option B may provide an estimate of the proportion of these students who are guessing.

Item #	Strand	Specific Outcome	Item Complexity	Item Description
39	SS	4	Low	Use information from a scaled diagram to determine the distance between two points on the diagram.

	% of Student Responses (*Correct)			
	A	**B**	**C***	**D**
Performance of Students Achieving the Standard of Excellence	3.1	5.5	83.8	7.6
Performance of Students Achieving the Acceptable Standard	11.2	19.2	54.8	14.4

The star on the map below represents the capital city of a country. Maria uses the map to locate a city that is 225 km from the capital.



39. Which of the cities on the map above is the city located by Maria?

A. City 1

B. City 2

C. City 3

D. City 4

Commentary:

Over four-fifths of the students who achieved the standard of excellence were successful on this item, while just over half of the students who achieved the acceptable standard were successful. Three-step items such as this, where students have to complete a set of measurements using centimetres, determine the scale of the drawing, and convert the centimetre measurements to actual distances using the map scale, are relatively straightforward for students who achieved the standard of excellence, but are difficult for students who achieved the acceptable standard.

Of the students who achieved the standard of excellence but answered the item incorrectly, option A was slightly less popular than the other two options. Option A could be rejected by a visual estimate, but precise measurements would be needed to distinguish among the other three options. The number of incorrect answers could be an estimate of the

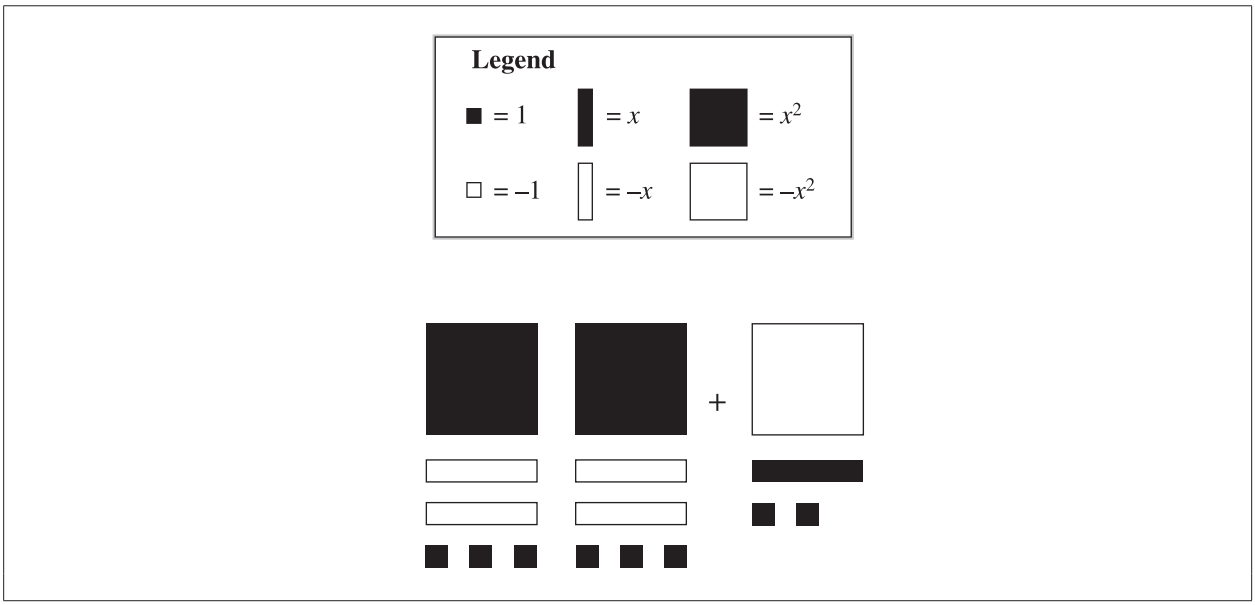
number of imprecise measurements or an indication of the amount of guessing; the evidence is consistent with both explanations.

Likewise, for the students who achieved the acceptable standard but answered the item incorrectly, the patterns of incorrect responses are similar to the patterns of responses for students at the standard of excellence.

The following four items, from all four strands, illustrate significant performance differences between students who achieved the acceptable standard and those who were below the acceptable standard.

Item #	Strand	Specific Outcome	Item Complexity	Item Description
7	PR	6	Moderate	Match a given polynomial expression that is represented in symbolic form to its equivalent pictorial representation.

	% of Student Responses (*Correct)			
	A*	B	C	D
Performance of Students Achieving the Acceptable Standard	91.5	4.4	2.7	1.4
Performance of Students Below the Acceptable Standard	60.7	16.0	15.7	7.3



7. Which of the following expressions represents the solution to the model shown above?

A. $x^2 - 3x + 8$
B. $x^2 + 3x - 8$
C. $-x^2 - 5x + 8$
D. $-x^2 + 5x - 8$

Commentary:

Over nine-tenths of students who achieved the acceptable standard were successful on this item, while only three-fifths of the students who did not achieve the acceptable standard were successful. Translation-type items, where diagrams are directly translated into symbols, are straightforward for students who achieved the acceptable standard, but are more difficult for students who did not achieve the acceptable standard.

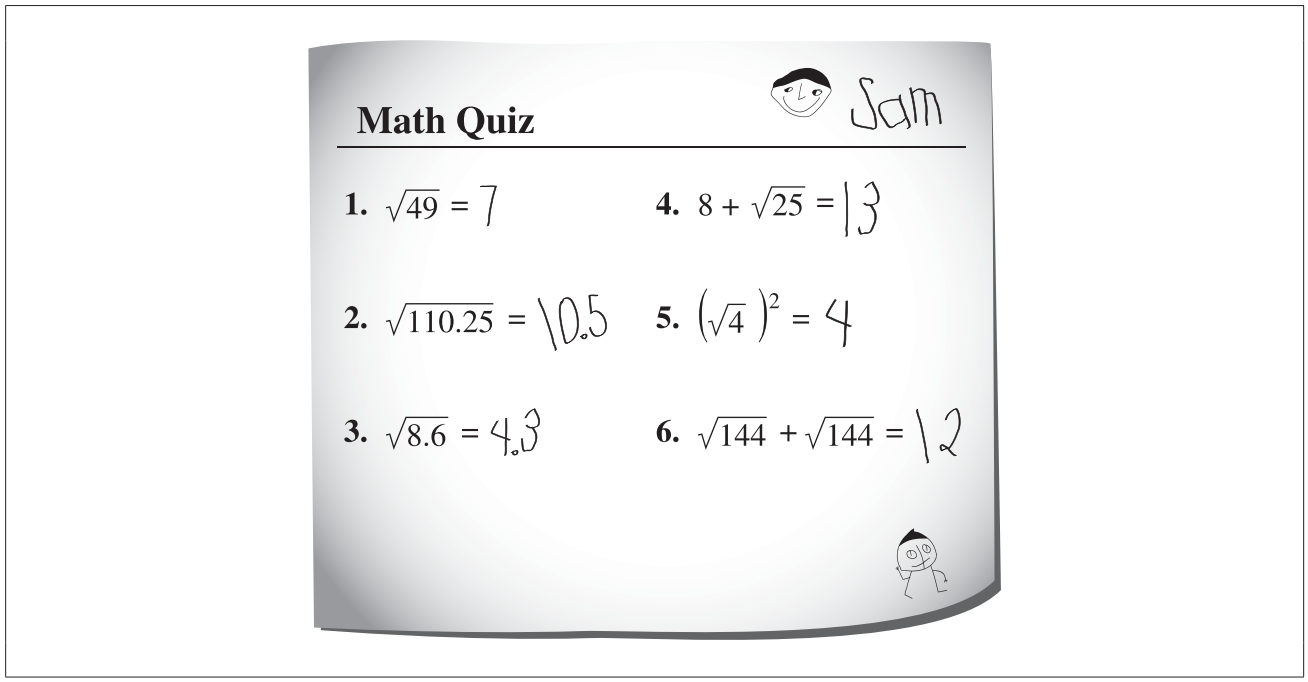
Of the few students who achieved the acceptable standard but answered the item incorrectly, option D was slightly less popular than the other two options. This option, chosen by only one-sixth of these students, meant that the student made errors in both coefficients and in the constant term. Option B, chosen by half of these students, could

be explained solely in terms of sign errors in x and in the constant term, with the coefficient of x^2 being correct. Option C, chosen by one-third of these students, could be explained solely in terms of a sign error in x^2 and an addition error in x, with the constant term being correct.

For the majority of students who did not achieve the acceptable standard and answered the item incorrectly, the pattern of errors was similar to that of students who achieved the acceptable standard.

Item #	Strand	Specific Outcome	Item Complexity	Item Description
13	N	5	High	Determine the square roots of a given set of positive rational numbers.

	% of Student Responses (*Correct)			
	A	B	C*	D
Performance of Students Achieving the Acceptable Standard	2.3	7.0	85.8	4.9
Performance of Students Below the Acceptable Standard	9.9	20.0	50.5	19.4



13. What percentage of the questions did Sam answer correctly?

A. 33%

B. 50%

C. 67%

D. 83%

Commentary:

Over four-fifths of students who achieved the acceptable standard were successful on this item, while only half of the students who did not achieve the acceptable standard were successful. Verification-type items, where short calculations are checked numerically for accuracy, are straightforward for students who achieved the acceptable standard, but are quite difficult for students who did not achieve the acceptable standard.

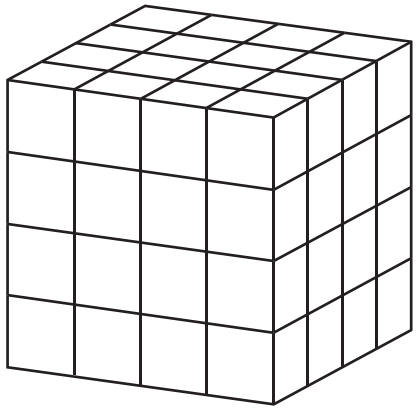
Of the students who achieved the acceptable standard but answered the item incorrectly, about half chose option B, which indicates that one of the correct calculations, probably calculation 4 or calculation 5, was marked as incorrect. Both of these require careful use of order of operations, especially if a four-function calculator is used. About one-third of these students chose option D, which indicates that one of the incorrect calculations, probably calculation 6, was marked as correct. Less than one-fifth of these students chose option A, apparently in the belief that only the direct taking of a single square root, with no other operations, can lead to a correct answer.

For the majority of students who did not achieve the acceptable standard and answered the item incorrectly, the pattern of errors was very similar to that of students who achieved the acceptable standard.

Item #	Strand	Specific Outcome	Item Complexity	Item Description
32	N	1	Moderate	Match the value represented by a given 3-D object to the correct repeated multiplication expression and power.

	% of Student Responses (*Correct)			
	A	B	C	D*
Performance of Students Achieving the Acceptable Standard	15.9	5.0	4.2	74.8
Performance of Students Below the Acceptable Standard	23.0	16.6	15.2	44.6

The cubes in the 3-D object shown below represent a repeated multiplication and a power.



32. Which of the following rows identifies the repeated multiplication and the power that the 3-D object represents?

Row	Repeated Multiplication	Power
A.	$3 \times 3 \times 3 \times 3$	3^4
B.	$3 \times 3 \times 3 \times 3$	4^3
C.	$4 \times 4 \times 4$	3^4
D.	$4 \times 4 \times 4$	4^3

Commentary:

Approximately three-quarters of students who achieved the acceptable standard were successful on this item, while just under half of the students who did not achieve the acceptable standard were successful. Translation-type items, where concepts are to be translated from one representation to another, are straightforward for students who achieved the acceptable standard, but are difficult for students who did not achieve the acceptable standard.

Of the students who achieved the acceptable standard but answered the item incorrectly, two-thirds chose option A, showing that they were able to recognize the base in a power expression, even though they were unable to recognize the key dimension (cube length) as 4 units, not 3 units. Options B and C, showing an inability to recognize the key dimension of cube length, were chosen by one-third of these students, with options B and C each being chosen by one-sixth of these students.

For the majority of students who did not achieve the acceptable standard and answered the item incorrectly, the pattern of errors was different from that of students who achieved the acceptable standard. Only two-fifths of these students chose option A, recognizing the base in the power expression. Options B and C, showing an inability to recognize the key dimension of cube length, were chosen by nearly two-thirds of these students, with options B and C each being chosen by almost one-third of these students.

Item #	Strand	Specific Outcome	Item Complexity	Item Description
NR 3	PR	1	High	Solve a problem by creating and solving a linear equation that represents a given context.

	% of Student Responses (*Correct)
Performance of Students Achieving the Acceptable Standard	71.7
Performance of Students Below the Acceptable Standard	18.6

Rahim plants a tree in his yard and records its initial height. The height of the tree at the end of the first growing season is 1.3 times its initial height. In the second growing season, the tree grows 14.5 cm to reach a total height of 71.7 cm.

Numerical Response

3. What was the **initial** height of the tree in centimetres?

Answer: _____ cm

(Record your answer in the numerical-response section on the answer sheet.)

Commentary:

Nearly three-quarters of the students who achieved the acceptable standard were successful on this item, while less than one-fifth of the students who did not achieve the acceptable standard were successful. Two-step items such as this, where students have to translate a verbal representation into a linear equation and then solve the two-step linear equation, are moderately easy for students who achieved the acceptable standard, but are extremely difficult for students who did not achieve the acceptable standard. The difficulty of 71.7% on this item for the group of students who achieved the acceptable standard, when contrasted with the difficulty of 18.6% for the group of students who did not achieve the acceptable standard on the complete test, indicates one type of item that can clearly distinguish students at or above the acceptable standard from students below the acceptable standard.

Sometimes numerical-response items can give indications of common misconceptions, but in this case there were over 1 300 different incorrect answers, many of which were given by only one or two students. The only common incorrect answers that were logical, the 57.2 cm height after the first growing season and the 55.9 cm height calculated by subtracting 1.3 from 57.2 rather than dividing it by 1.3, were each provided by about 3% of students. No other answer, logical or otherwise, drew more than 0.8%. This wide range of responses can be attributed to the difficulty students experienced in translating the context into a linear equation; if the translation part of the problem had been completed correctly, the range of possible responses would have been very restricted.

Achievement Testing Program Support Documents

The Alberta Education website contains several documents that provide valuable information about various aspects of the achievement testing program. To access these documents, go to the Alberta Education website at education.alberta.ca. From the home page, follow the path *Teachers > Provincial Testing > Provincial Achievement Tests (PAT)*, and then click on one of the specific links to access the following documents.

Achievement Testing Program *General Information Bulletin*

The *General Information Bulletin* is a compilation of several documents produced by Alberta Education and is intended to provide superintendents, principals, and teachers with easy access to information about all aspects of the achievement testing program. Sections in the bulletin contain information pertaining to schedules and significant dates; security and test rules; test administration directives, guidelines, and procedures; calculator and computer policies; test accommodations; test marking and results; field testing; resources and web documents; forms and samples; and Assessment Sector contacts.

Subject Bulletins

At the beginning of each school year, subject bulletins are posted on the Alberta Education website for all achievement test subjects for grades 6 and 9. Each bulletin provides descriptions of assessment standards, test design and blueprinting, and scoring guides (where applicable) as well as suggestions for preparing students to write the tests and information about how teachers can participate in test development activities.

Examples of the Standards for Students' Writing

For achievement tests in grades 6 and 9 English Language Arts and Français/French Language Arts, writing samples have been designed to be used by teachers and students to enhance students' writing and to assess this writing relative to the standards inherent in the scoring guides for the achievement tests. The exemplars documents contain sample responses with scoring rationales that relate student work to the scoring categories and scoring criteria.

Previous Achievement Tests and Answer Keys

All January achievement tests (parts A and B) for Grade 9 semestered students are secured and must be returned to Alberta Education. All May/June achievement tests are secured except Part A of grades 6 and 9 English Language Arts and Français/French Language Arts. Unused or extra copies of only these Part A tests may be kept at the school after administration. Teachers may also use the released items and/or tests that are posted on the Alberta Education website.

Parent Guides

Each school year, versions of the *Alberta Provincial Achievement Testing Parent Guide* for grades 6 and 9 are posted on the Alberta Education website. Each guide presents answers to frequently asked questions about the achievement testing program as well as descriptions of and sample questions for each achievement test subject.

Involvement of Teachers

Teachers of grades 6 and 9 are encouraged to take part in activities related to the achievement testing program. These activities include item development, test validation, field testing, and marking. In addition, arrangements can be made through the Alberta Regional Professional Development Consortia for teacher in-service workshops on topics such as Interpreting Achievement Test Results to Improve Student Learning.