ALBERTA EDUCATION

Mathematics 30–1 Practice Test

Diploma Examinations Program **2022**

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Teachers	✓	of Mathematics 30–1
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Others		

Mathematics 30–1 Practice Test

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Contents

Please note that if you cannot access one of the direct website links referred to in this document, you can find diploma examination-related materials on the Alberta Education website.

Introduction

The questions in this booklet are a sample set of questions for Mathematics 30–1. Teachers may wish to use these questions in a variety of ways to help students develop and demonstrate an understanding of the concepts described in the Mathematics 30–1 Program of Studies. This material, along with the Program of Studies, Information Bulletin, and Assessment Standards and Exemplars, can provide insights that assist with decisions about instructional planning.

These questions are available in both English and French.

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To call toll-free from outside Edmonton, dial 310-0000.

Additional documents

The Provincial Assessment Sector supports the instruction of Mathematics 30–1 with the following documents available online on the Writing diploma exams web page.

- *Mathematics 30-1 Information Bulletin*
- *Mathematics 30-1 Assessment Standards and Exemplars*
- *Mathematics 30-1 Released Items*
- *Mathematics 30-1 Written-Response Information*

Mathematics 30–1 Practice Test

Use the following information to answer numerical-response question 1.

The mapping notation $(x, y) \rightarrow (x - 2, \ -y + 5)$ is used to describe the transformation of the function $y = f(x)$ into the function $y = g(x)$.

Possible Transformations

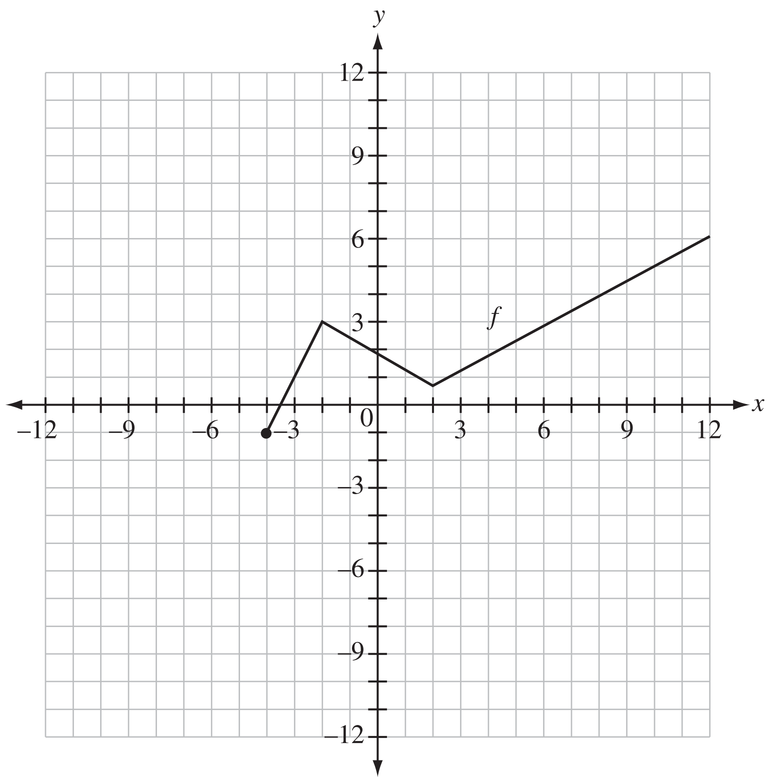
1	Reflection in the *x*-axis
2	Reflection in the *y*-axis
3	Translation 5 units right
4	Translation 5 units left
5	Translation 2 units right
6	Translation 2 units left
7	Translation 5 units up
8	Translation 5 units down
9	Translation 2 units up
0	Translation 2 units down

Numerical Response

1. In correct order, the transformations above that would transform $y = f(x)$ into $y = g(x)$ are numbered ______, ______, and ______. (There is more than one correct answer.)

Use the following information to answer question 1.

The graph of $y = f(x)$, shown below, has the domain $\{x \mid x \geq -4, x \in R\}$.



1. The domain of the graph of $y = f(-x + 4)$ is

A. $\{x \mid x \leq 8, x \in R\}$

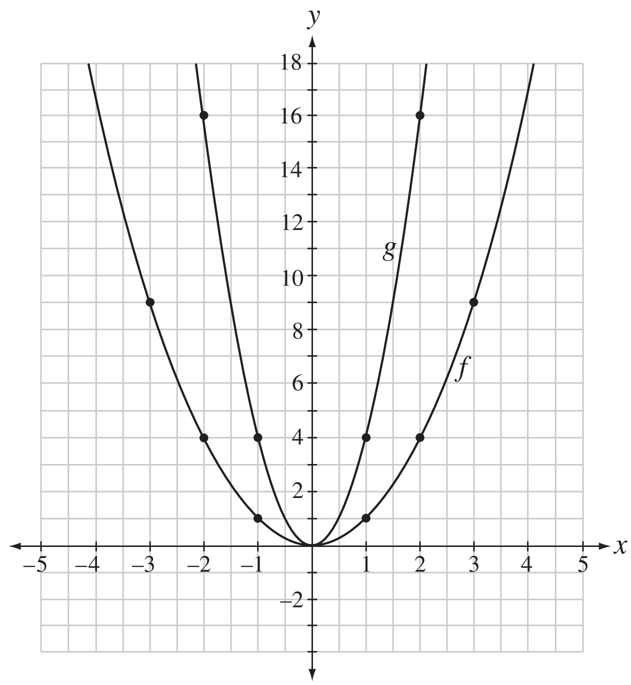
B. $\{x \mid x \geq -8, x \in R\}$

C. $\{x \mid x \leq 0, x \in R\}$

D. $\{x \mid x \geq 0, x \in R\}$

Use the following information to answer question 2.

The graph of $y = f(x)$ is transformed into the graph of $y = g(x)$, as shown below.



2. A possible equation for $g(x)$ is

A. $g(x) = 2f(x)$

B. $g(x) = f(2x)$

C. $g(x) = \frac{1}{2}f(x)$

D. $g(x) = f\left(\frac{1}{2}x\right)$

Use the following information to answer question 3.

The point $A(2, 1)$ is on the graph of $y = f(x)$. The graph is stretched horizontally about the y-axis and then translated so that the new graph passes through the corresponding point $A'(8, 1)$. The equation of the new function can be written in the form $y = f\big(m(x - 2)\big)$.

3. The value of ***m*** is

A. $\frac{1}{5}$

B. $\frac{1}{3}$

C. 3

D. 5

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4. Given the function $f(x) = 3^{(x-a)} + b$, $a \neq 0$, the domain of the inverse of $f(x)$ is

A. $\{x \mid x > 0, x \in R\}$

B. $\{x \mid x > 3, x \in R\}$

C. $\{x \mid x > a, x \in R\}$

D. $\{x \mid x > b, x \in R\}$

Numerical Response

2. Given that $C = B^2$ and $B^2 = A^5$, where $A, B, C > 0$, $A \neq 1$, the value of $\log_A(BC)$, to the nearest tenth, is __________.

5. An expression equivalent to $-\log x - \log y$ is

A. $\dfrac{-\log x}{\log y}$

B. $-\log\left(\dfrac{x}{y}\right)$

C. $\log\left(\dfrac{1}{xy}\right)$

D. $\dfrac{1}{\log x \log y}$

Numerical Response

3. The expression $\log_b b^2 - 5\log_b c + 4\log_b\left(bc^3\right)$, where $b > 1$ and $c > 1$, can be written in the form $m + n\log_b c$.

The single-digit value of ***m*** is __________. (Record in the **first** column)

The single-digit value of ***n*** is __________. (Record in the **second** column)

Use the following information to answer question 6.

Three of the following functions have the same vertical asymptote.

1 $f(x) = \log_2(x - 3) + 6$

2 $g(x) = \log_2(2x - 6) + 3$

3 $h(x) = \dfrac{2x^2 + 6x}{x^2 - 3x}$

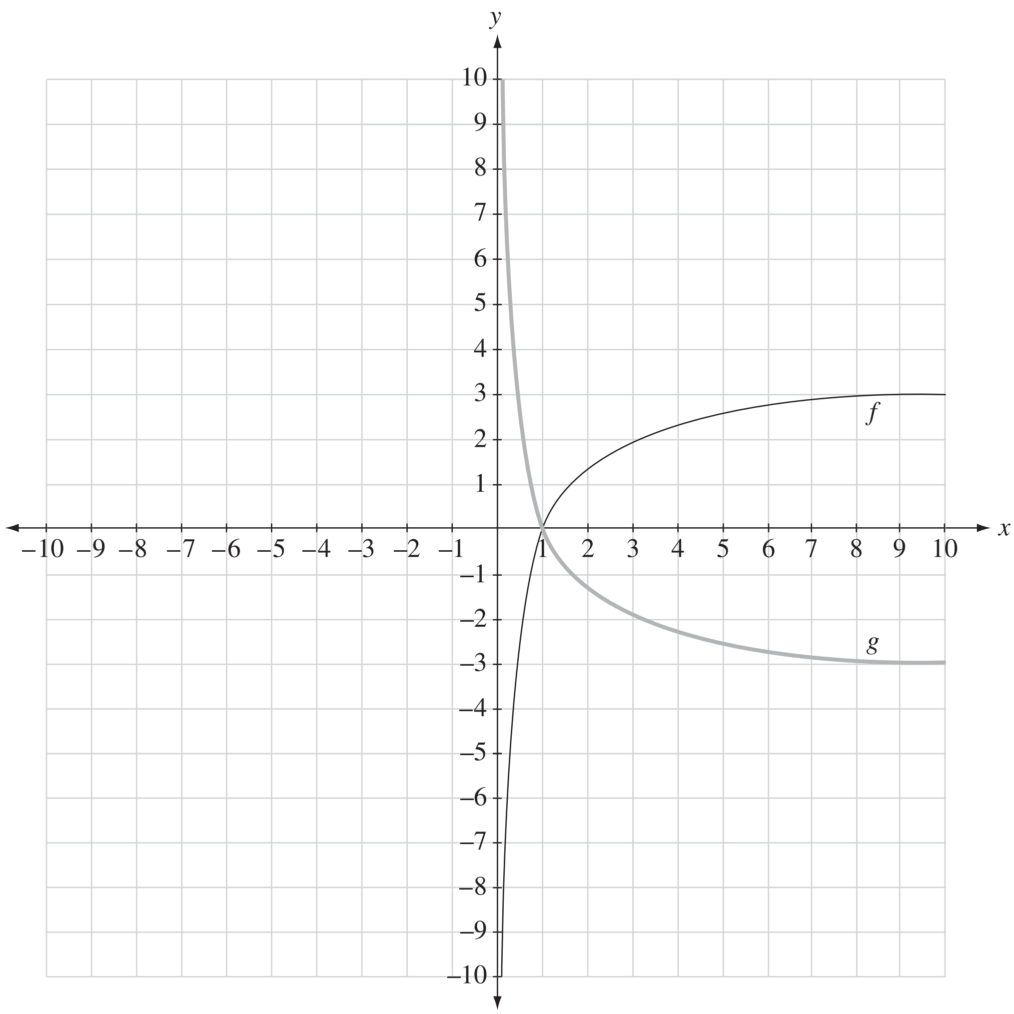
4 $k(x) = \dfrac{x - 3}{2x^2 - 6x}$

6. The function whose graph does **not** have the same vertical asymptote as the other graphs is Function

A. 1
B. 2
C. 3
D. 4

Use the following information to answer question 7.

The graphs of $f(x) = \log_a x$ and $g(x) = \log_b x$ are shown below.



7. When the equations of the above functions are compared, which of the following statements is true?

A. $a < b$

B. $b > 1$

C. $a > b$

D. $a < 1$

Use the following information to answer question 8.

Hala correctly solved the equation $\log_3(x + 3) + \log_3(x - 5) = 2$ using an algebraic process. She determined that one of the possible roots is extraneous.

8. The possible root that is extraneous is

A. $x = -6$

B. $x = -5$

C. $x = -4$

D. $x = -3$

Use the following information to answer question 9.

The apparent brightness of stars is expressed in terms of magnitude, M, on a numerical scale that increases as the brightness decreases, as given by the formula

$$M = 6 - 2.5\log\left(\frac{L}{L_0}\right)$$

where L is the light flux of a given star and L_0 is the light flux of the dimmest star visible to the unaided human eye.

9. How many times greater is the light flux from a star with a magnitude of –1.5 than the light flux from a star with a magnitude of 3.5?

A. 100

B. 1 000

C. 10 000

D. 100 000

Use the following information to answer question 10.

Four statements about the polynomial function $P(x) = x^3 - 2x^2 - 13x - 10$ are shown below.

Statement 1 When $P(x)$ is divided by $(x + 1)$, the quotient is $x^2 - 3x - 10$.

Statement 2 $P(x)$ has a factor of $(x - 1)$.

Statement 3 $P(-2) = 0$, so $(x - 2)$ is a factor of $P(x)$.

Statement 4 $P(-3) = -16$, so -16 is the remainder when $P(x)$ is divided by $(x + 3)$.

10. The two statements about $P(x)$ that are correct are statements

A. 1 and 3
B. 1 and 4
C. 2 and 3
D. 2 and 4

Use the following information to answer question 11.

Michael graphed the function $y = a(x - b)^2(x + c)$, where $a, b, c > 0$.

11. If Michael changes the function so that $a < 0$, and then compares the new graph to the graph of the original function, then there will be a change in the

A. domain
B. range
C. x-intercepts
D. y-intercept

Use the following information to answer question 12.

The graph of the function $P(x) = a(x - r)(x - 1)^2(x - 4)$ passes through the point (0, 6).

12. The value of $\boldsymbol{a}$ in terms of $\boldsymbol{r}$ is

A. $a = \dfrac{3}{2r}$

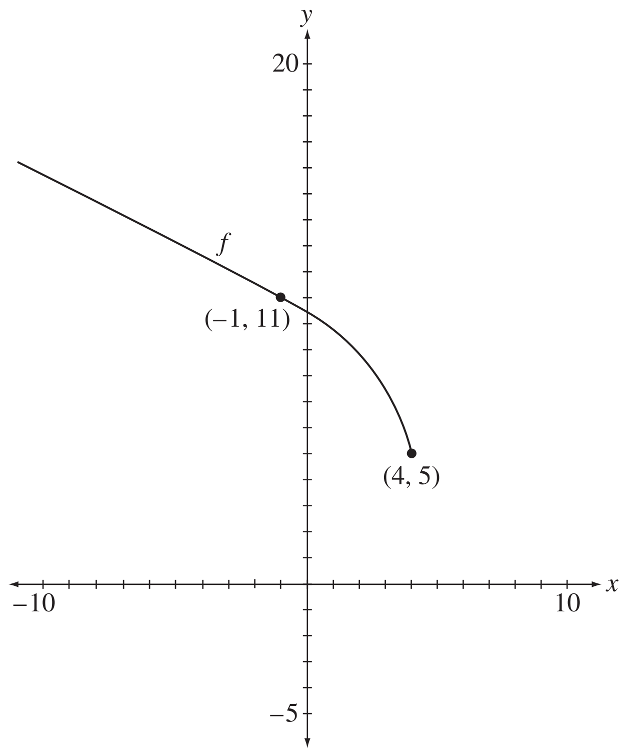
B. $a = -\dfrac{3}{2r}$

C. $a = \dfrac{2}{3r}$

D. $a = -\dfrac{2}{3r}$

Use the following information to answer numerical-response question 4.

The graph of $y = f(x)$ is shown below.



The equation for the graph of $y = f(x)$ can be written in the form $f(x) = a\sqrt{-(x-h)} + k$.

Numerical Response

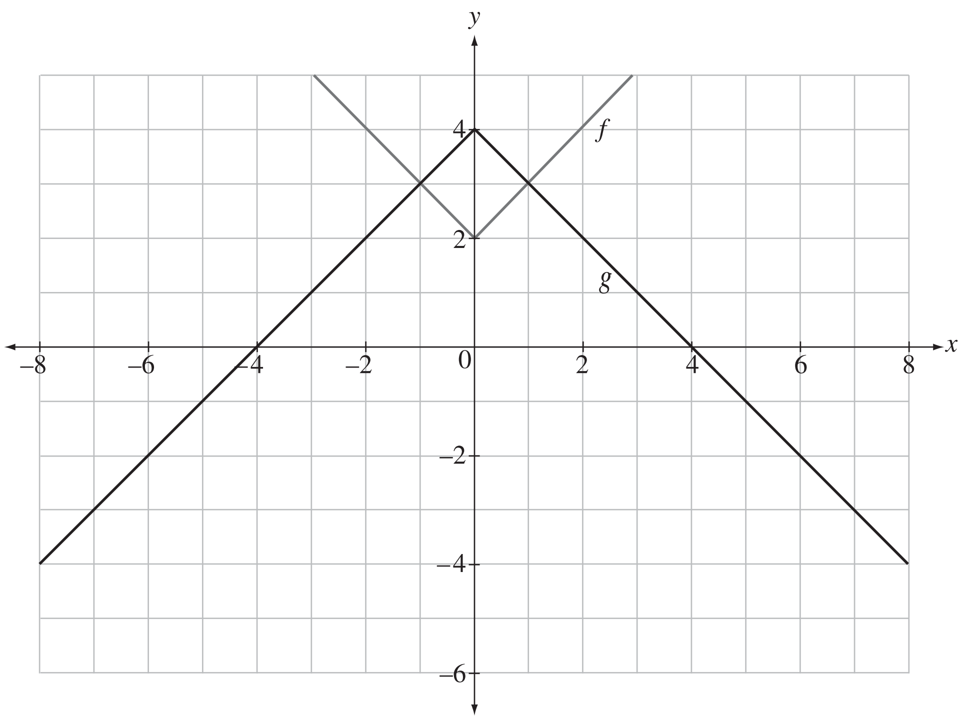
4. The value of $\boldsymbol{a}$, to the nearest hundredth, is __________.

13. Given $f(x) = 8 - 3x$, $g(x) = \left|\frac{1}{2}x - 5\right|$, and $h(x) = \log_{\frac{1}{2}} x$, the value of $(f \circ g \circ h)(16)$ is

A. –13
B. –5
C. 29
D. 480

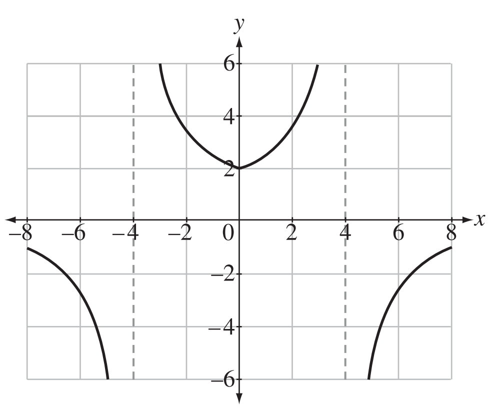
Use the following information to answer question 14.

The graphs of $y = f(x)$ and $y = g(x)$ are shown below.

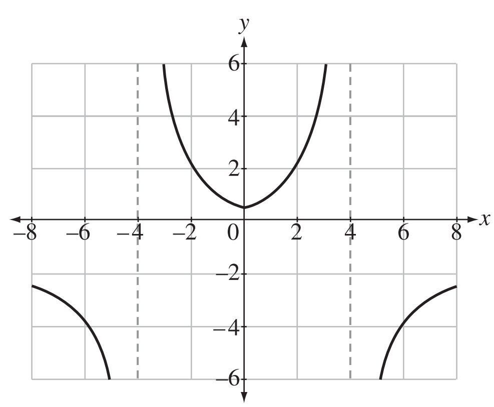


14. Which of the following graphs represents $y = \dfrac{f(x)}{g(x)}$?

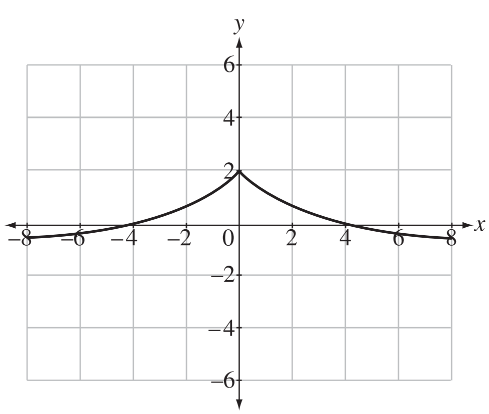
A.



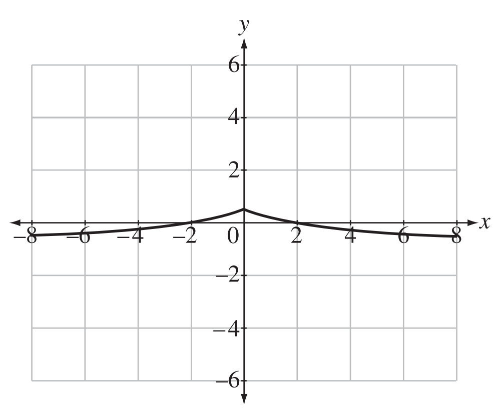
B.



C.

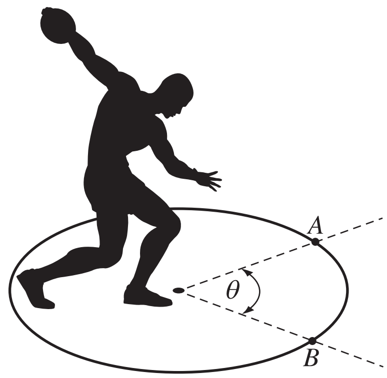


D.



Use the following information to answer question 15.

Track and field athletes participating in the discus competition must stay in the throwing circle, which has a diameter of 2.5 m, and their discus must land within the boundary lines. The boundary lines are set from the centre of the circle through two points, A and B, on the circumference of the circle. The length of the arc between points A and B is 87 cm.



Note: The diagram is **not** drawn to scale.

15. Angle θ, to the nearest degree, is

A. 20°
B. 35°
C. 40°
D. 82°

Use the following information to answer numerical-response question 5.

Points $A\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ and $B\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ lie on the unit circle.

Numerical Response

5. If Point C (0, 0) is the centre of the unit circle, then the measure, to the nearest hundredth of a radian, of the smaller Angle ACB is __________ rad.

Use the following information to answer question 16.

Angle θ is drawn in standard position on a unit circle. The point $P\left(x, -\frac{8}{17}\right)$, where $x > 0$, lies on the terminal arm of Angle θ.

16. What is the **exact** value of $\cot\theta$?

A. $-\frac{8}{\sqrt{353}}$

B. $-\frac{\sqrt{353}}{8}$

C. $-\frac{8}{15}$

D. $-\frac{15}{8}$

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17. If $\csc\left(\frac{4\pi}{3}\right) + k = \cot\left(\frac{8\pi}{3}\right)$, then the **exact** value of ***k*** is

A. $-\sqrt{3}$

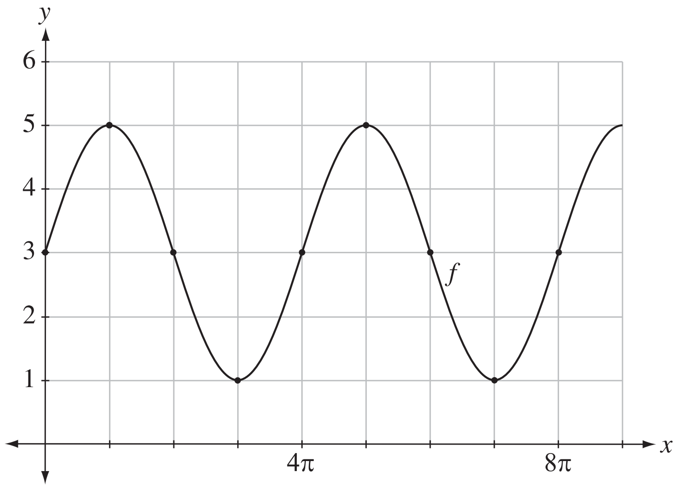
B. $-\frac{1}{\sqrt{3}}$

C. $\frac{1}{\sqrt{3}}$

D. $\sqrt{3}$

Use the following information to answer question 18.

The graph of the function $f(x) = a\cos\left[b(x - c)\right] + d$, where $a > 0$, is shown below.



18. An equation for $f(x)$ with the correct values for the parameters ***b*** and ***c*** is

A. $f(x) = a\cos\left[\frac{1}{2}(x - \pi)\right] + d$

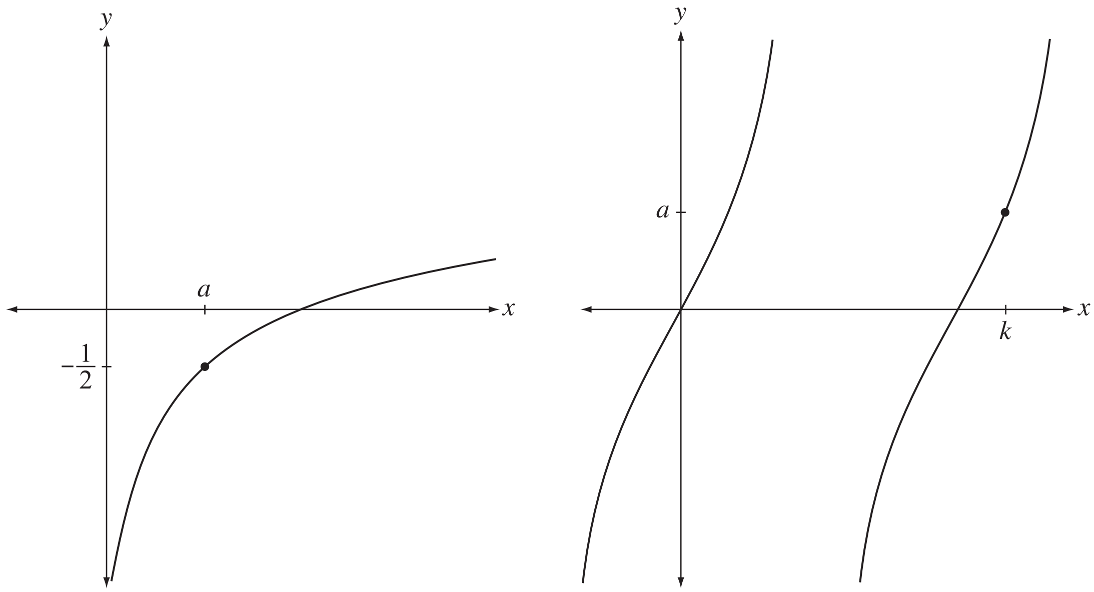
B. $f(x) = a\cos\left[\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right] + d$

C. $f(x) = a\cos[2(x - \pi)] + d$

D. $f(x) = a\cos\left[2\left(x - \frac{\pi}{2}\right)\right] + d$

Use the following information to answer question 19.

The graphs of $y = \log_3 x$ and $y = \tan x$ are shown below. The point $\left(a, -\frac{1}{2}\right)$ is marked on the graph of $y = \log_3 x$, and the point (k, a) is marked on the graph of $y = \tan x$.



19. The value of ***k*** is

A. 120°

B. 150°

C. 210°

D. 240°

Use the following information to answer numerical-response question 6.

Students are solving an equation of the form $(2\sin x - 2)(a\sin x + b) = 0$, where $a > 0$, $b > 0$, and $a > b$.

Numerical Response

6. If the domain is $0 \le x < 2\pi$, then the **number** of solutions to this equation is __________.

Use the following information to answer question 20.

Four students wrote the general solution to the equation $10\cos^2\theta = 5$ as shown below.

Sandy $\theta = \frac{\pi}{4} + n\pi,\ n \in I$

Noah $\theta = \frac{\pi}{4} + \frac{n\pi}{2},\ n \in I$

Luke $\theta = \frac{\pi}{4} + 2n\pi,\ n \in I,$ and $\theta = \frac{7\pi}{4} + 2n\pi,\ n \in I$

Jane $\theta = \frac{\pi}{4} + n\pi,\ n \in I,$ and $\theta = \frac{3\pi}{4} + n\pi,\ n \in I$

20. The two students who have stated a correct general solution are

A. Sandy and Luke
B. Sandy and Jane
C. Noah and Luke
D. Noah and Jane

Use the following information to answer question 21.

A student incorrectly simplified the expression $\left(\dfrac{\dfrac{2\cot x}{\csc^2 x}}{\cos^2 x + \sin^2 x}\right)$. The steps of the student's work are shown below.

Step 1 $\left(\dfrac{\dfrac{2\cos x}{\sin x}}{\dfrac{1}{\sin^2 x}}\right)\left(\dfrac{1}{\cos^2 x + \sin^2 x}\right)$

Step 2 $\left(\dfrac{2\cos x}{\sin x} \bullet \dfrac{\sin^2 x}{1}\right)\left(\dfrac{1}{1 + \sin^2 x + \sin^2 x}\right)$

Step 3 $(2\cos x \sin x)\left(\dfrac{1}{1 + 2\sin^2 x}\right)$

Step 4 $\dfrac{\sin(2x)}{\cos(2x)}$

Step 5 $\tan(2x)$

21. The student's **first** error was recorded in Step

A. 1

B. 2

C. 3

D. 4

Use the following information to answer numerical-response question 7.

Eden is booking a vacation package. She has the option of travelling by train or by plane. The train has only one type of ticket. If she takes the plane, she must choose a regular ticket or a first-class ticket. Regardless of whether she travels by train or by plane, she must make a meal choice during travel of chicken, beef, or vegetarian. At her destination, she must choose one of three different hotels.

Numerical Response

7. The number of different vacation packages that Eden can select is __________.

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22. The number of distinct arrangements using all 10 of the letters in the word **ACCESSIBLE** if the vowels, AEIE, must all be together is

A. 60 480
B. 15 120
C. 8 640
D. 2 160

Use the following information to answer question 23.

Josef is selecting 5 songs to be randomly played between games at a basketball tournament. Each of the 10 players on the team suggested a different song. The coach suggested 4 songs that are all different from those suggested by the players.

23. The number of different possible selections of songs that Josef can make that use exactly 2 of the coach's suggestions is

A. 8 640
B. 2 002
C. 720
D. 126

24. In the expansion of $(x - 2y)^8$, written in descending powers of x, the coefficient of the middle term is

A. 1 120
B. –1 120
C. 1 792
D. –1 792

Use the following information to answer numerical-response question 8.

In the expansion of the binomial $(x + a)^b$ written in descending powers of x, given $a, b \in N$, the second and third terms are $24x^7$ and $252x^6$, respectively.

Numerical Response

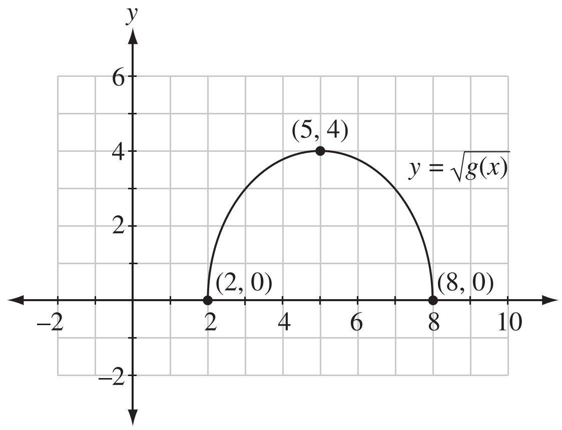
8. The values of $\boldsymbol{a}$ and $\boldsymbol{b}$ are, respectively, _____ and _____.

Written-response question 1 begins on the next page.

Use the following information to answer written-response question 1.

The graph of $h(x) = x^2$ is transformed into the graph of $y = g(x)$, whose equation can be written in the form $g(x) = a(x - h)^2 + k$.

The graph of $y = \sqrt{g(x)}$, shown on the right, has a maximum at the point (5, 4).



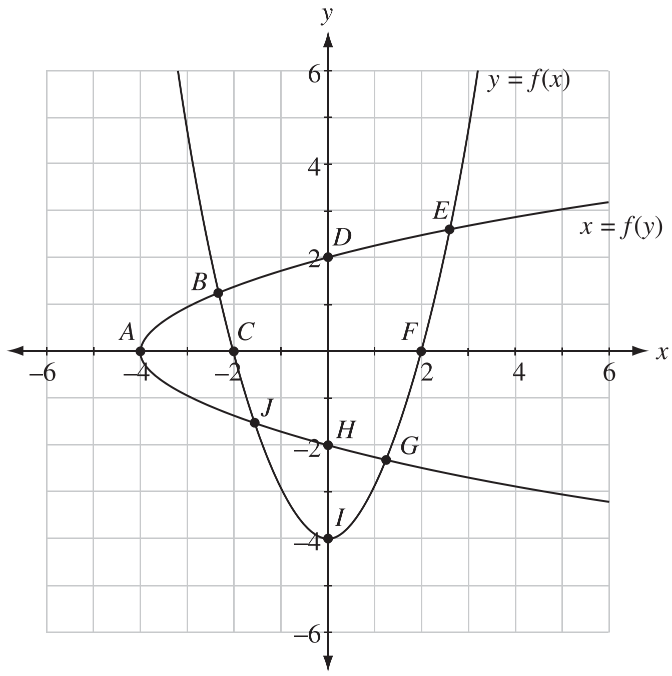
Written Response—5 Marks

1. **a.** **Sketch** the graph of $y = g(x)$ on the grid provided below, and clearly label the x-intercepts and maximum. **Describe** one possible sequence of transformations that could be applied to the graph of $y = h(x)$ to produce the graph of $y = g(x)$. **[3 marks]**

Transformations:

Use the following information to answer the next part of written-response question 1.

The graph of the quadratic function $y = f(x)$ is transformed to create the graph of $x = f(y)$. The graphs of $y = f(x)$ and $x = f(y)$ and 10 labelled points are shown below.

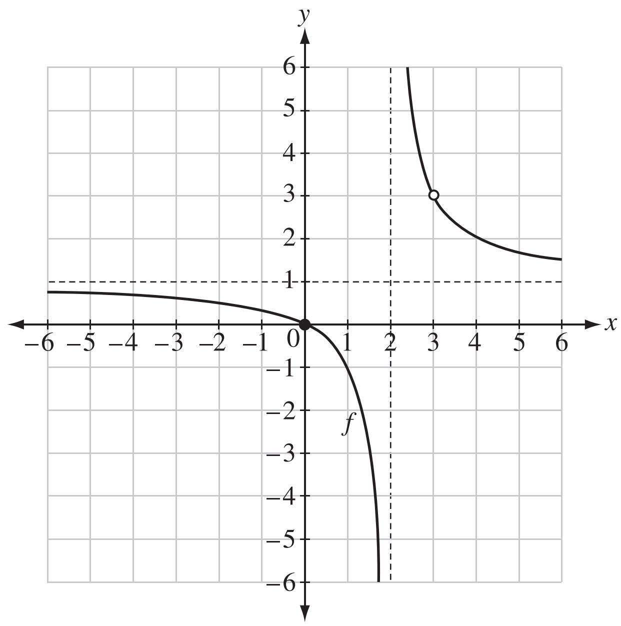


b. Identify the invariant points and **explain** why they are invariant. **[2 marks]**

Written-response question 2 begins on the next page.

Use the following information to answer written-response question 2.

The graph of $y = f(x)$ and the equation of $y = g(x)$ are shown below.



$$g(x) = \frac{x+3}{x-2}$$

Written Response—5 marks

2. **a.** **Compare** the intercepts, equations of the asymptotes, and domains of $y = f(x)$ and $y = g(x)$. **[3 marks]**

Use the following information to answer the next part of written-response question 2.

Betty is creating the function $h(x) = \dfrac{(x - a)(x - b)}{(x - c)}$ using the following values for a, b, and c:

$$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$$

Betty would like the graph of the function to have a vertical asymptote and two distinct x-intercepts, one of which is negative. She may use the provided values more than once.

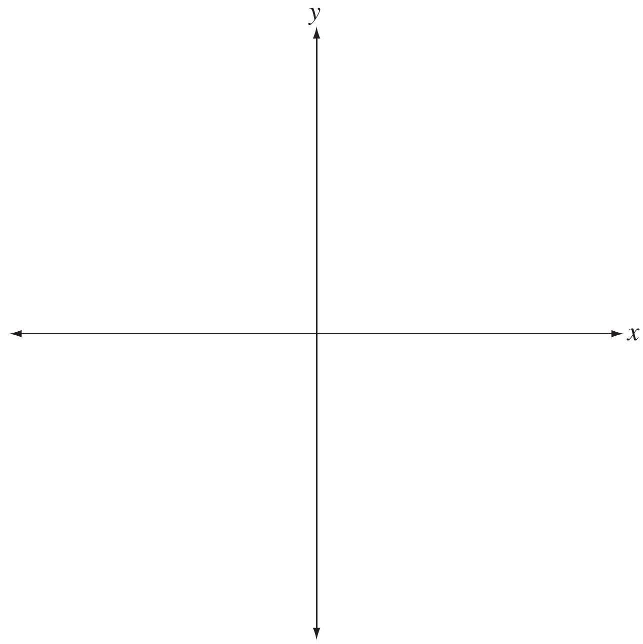
b. Provide an example of an equation Betty could use to represent the function, and **determine** the total number of different graphs that would meet Betty's criteria. **[2 marks]**

Written-response question 3 begins on the next page.

Written Response—5 Marks

3. **a.** **Determine** the value of Angle θ in standard position such that $\csc\theta = -\sqrt{2}$, where $540° \leq \theta \leq 630°$. **Sketch** and label Angle θ in standard position on the Cartesian plane below. **[2 marks]**

$\theta =$ ___________



Use the following information to answer the next part of written-response question 3.

The terminal arm of Angle β intersects the unit circle at Point $P\left(m, -\frac{\sqrt{3}}{4}\right)$, where $\sec\beta > 0$.

b. Algebraically determine the exact value of $\cos\left(\beta + \frac{\pi}{6}\right)$ in the form $\frac{\sqrt{a} + \sqrt{b}}{c}$, where $a, b, c \in N$. **[3 marks]**

Mathematics 30–1 Practice Test Answer Key

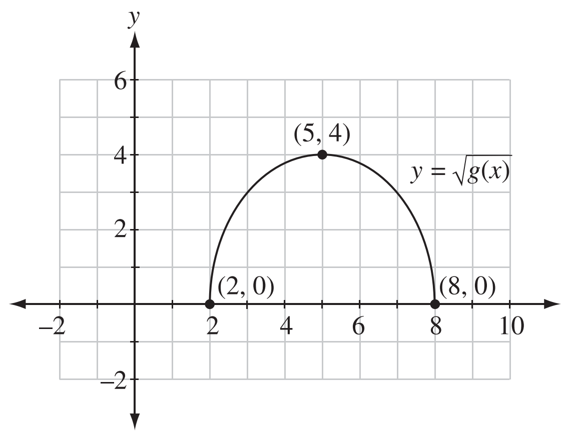
Question	Key
NR 1	167/176/617/816/861/681
MC 1	A
MC 2	B
MC 3	B
MC 4	D
NR 2	7.5
MC 5	C
NR 3	67
MC 6	D
MC 7	C
MC 8	C
MC 9	A
MC 10	B
MC 11	D
MC 12	A
NR 4	2.68
MC 13	A
MC 14	B
MC 15	C
NR 5	2.88
MC 16	D
MC 17	C
MC 18	A
MC 19	C
NR 6	3
MC 20	D
MC 21	B
NR 7	27
MC 22	B
MC 23	C
MC 24	A
NR 8	38
WR 1	See Sample Solution
WR 2	See Sample Solution
WR 3	See Sample Solution

Written-response Question 1 Sample Solution

Use the following information to answer written-response question 1.

The graph of $h(x) = x^2$ is transformed into the graph of $y = g(x)$, whose equation can be written in the form $g(x) = a(x - h)^2 + k$.

The graph of $y = \sqrt{g(x)}$, shown on the right, has a maximum at the point (5, 4).



Written Response—5 Marks

1. **a.** **Sketch** the graph of $y = g(x)$ on the grid provided below, and clearly label the x-intercepts and maximum. **Describe** one possible sequence of transformations that could be applied to the graph of $y = h(x)$ to produce the graph of $y = g(x)$. **[3 marks]**

A possible solution to part a

The graph of $y = g(x)$ will have the same x-intercepts as the graph of $y = \sqrt{g(x)}$ but a maximum point at (5, 16).

The graph of $g(x)$ will also extend below the x-axis.

A possible equation of $y = g(x)$:

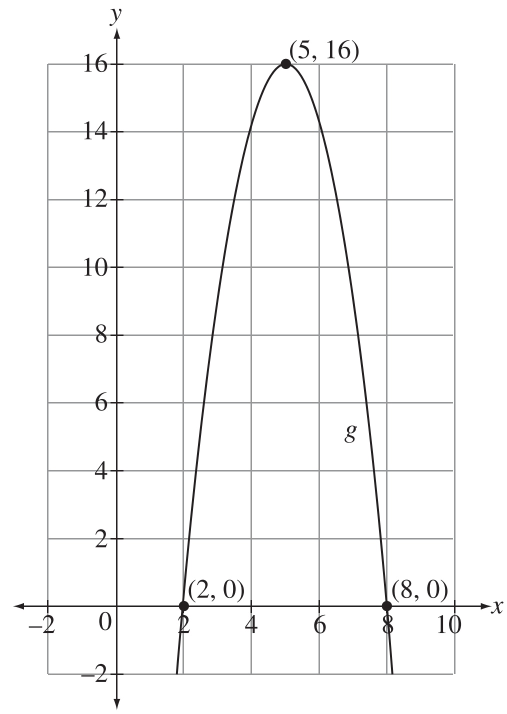
$$g(x) = a(x - 5)^2 + 16$$

$$0 = a(8 - 5)^2 + 16$$

$$-16 = a(3)^2$$

$$a = -\frac{16}{9}$$

$$g(x) = -\frac{16}{9}(x - 5)^2 + 16$$

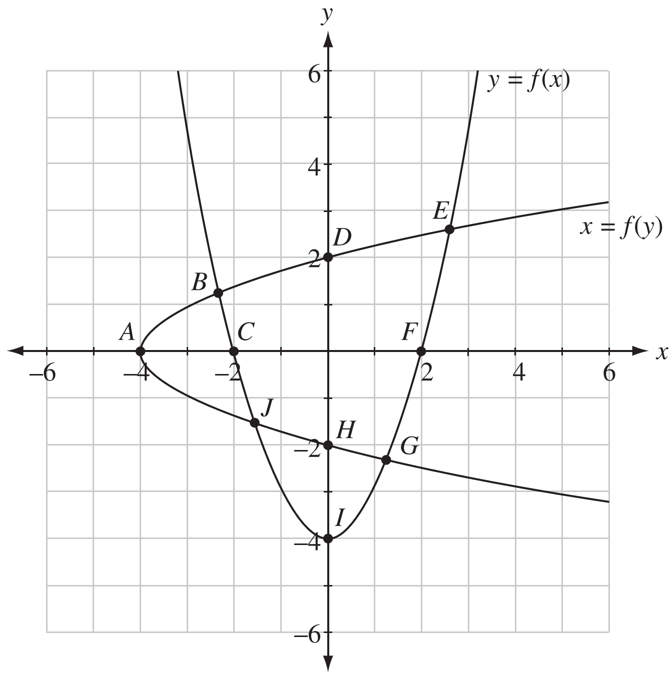


A sequence of transformations that could be applied to $y = h(x)$ to result in $y = g(x)$ is

- a vertical stretch about the x-axis by a factor of $\frac{16}{9}$
- a reflection in the x-axis
- a horizontal translation 5 units right
- a vertical translation 16 units up

Use the following information to answer the next part of written-response question 1.

The graph of the quadratic function $y = f(x)$ is transformed to create the graph of $x = f(y)$. The graphs of $y = f(x)$ and $x = f(y)$ and 10 labelled points are shown below.



b. Identify the invariant points and **explain** why they are invariant. **[2 marks]**

A possible solution to part b

The invariant points are labelled on the graph as ***E*** and ***J***.

First Possible Explanation

The graph of the inverse of a relation can be created by reflecting the graph of $y = f(x)$ in the line $y = x$. Any points on the graph of $y = f(x)$ that intersect with the reflection line $y = x$ would remain unchanged and therefore be invariant.

Second Possible Explanation

When a function is transformed into its corresponding inverse relation, the *x*-coordinate of each point becomes the *y*-coordinate of the corresponding point and the *y*-coordinate of each point becomes the *x*-coordinate of the corresponding point. This means that invariant points will only exist when the value of *x* is equal to the value of *y*.

Scoring guide for written-response question 1

PART A

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	The response does not provide any correct descriptions of the transformations or a sketch of the related graph.
0.5		
1	In the response, the student demonstrates minimal mathematical understanding of the problem by applying an appropriate strategy or some relevant mathematical knowledge to complete initial stages of a solution.	In the response, the student • provides a complete description of the transformations OR • sketches a graph with 1 correct characteristic and describes 2 correct transformations OR • sketches a graph with 2 correct characteristics
1.5		
2	In the response, the student demonstrates good mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student • sketches the correct graph of $y = g(x)$ OR • sketches a graph with 3 correct characteristics and describes 3 correct transformations OR • sketches a graph with 2 or 3 correct characteristics and describes the correct corresponding transformations
2.5		
3	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • sketches the correct graph of $y = g(x)$ and describes a correct sequence of transformations

A complete and correct sketch includes

- the correct graph shape
- a vertex at (5, 16) and *x*-intercepts at (2, 0) and (8, 0)
- correct domain and range (graph extends below the *x*-axis)
- appropriately scaled *x*- and *y*-axes

PART B

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	The response does not contain any relevant information about invariant points.
0.5		
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student • identifies the correct invariant points OR • provides a complete explanation about why points are invariant OR • identifies a list of invariant points that include J or E, but one of the points is missing or there are at most two additional points listed, and provides a partial explanation about why the points are invariant
1.5		
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • correctly identifies the invariant points and provides a complete explanation about why the points are invariant

A complete explanation includes either

• indicating that the inverse is a reflection on the line $y = x$ AND acknowledging that the invariant points are located on the reflection line $y = x$

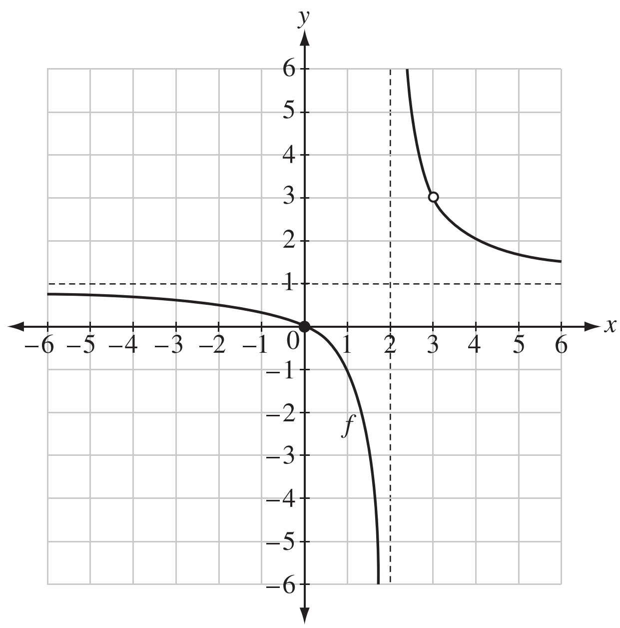
OR

• indicating that the inverse is created by switching the *x* and *y* coordinates AND acknowledging that the invariant points have equal *x* and *y* coordinates

Written-response Question 2 Sample Solution

Use the following information to answer written-response question 2.

The graph of $y = f(x)$ and the equation of $y = g(x)$ are shown below.



$$g(x) = \frac{x+3}{x-2}$$

Written Response—5 marks

2. **a.** **Compare** the intercepts, equations of the asymptotes, and domains of $y = f(x)$ and $y = g(x)$. **[3 marks]**

A possible solution to part a

For $g(x)$: $g(x)=\dfrac{x+3}{x-2} \rightarrow g(x)=\dfrac{x-2+5}{x-2}$

$$g(x)=\frac{5}{x-2}+\frac{x-2}{x-2}$$

$$g(x)=\frac{5}{x-2}+1,\ \text{where}\ \ x\neq 2$$

The graph of $y=g(x)$, when compared to $y=\dfrac{1}{x}$, has been translated 1 unit up so the horizontal asymptote is $y=1$. The vertical asymptote is $x=2$, which means the domain is $\{x \mid x\neq 2, x\in R\}$.

x-intercept: $0=\dfrac{x+3}{x-2}$

$0=x+3$

$x=-3$

y-intercept: $y=\dfrac{0+3}{0-2}$

$y=-\dfrac{3}{2}$

The graphs of $y=f(x)$ and $y=g(x)$ have different *x*- and *y*-intercepts. The graph of $y=f(x)$ has an *x*-intercept at $(0, 0)$ and a *y*-intercept at $(0, 0)$. The graph of $y=g(x)$ has an *x*-intercept at $(-3, 0)$ and a *y*-intercept at $\left(0, -\dfrac{3}{2}\right)$.

Both graphs have a horizontal asymptote at $y=1$ and a vertical asymptote at $x=2$, but the graph of $y=f(x)$ also has a point of discontinuity at $x=3$, meaning the domains will be different. The domain of $y=g(x)$ is $\{x \mid x\neq 2, x\in R\}$ and the domain of $y=f(x)$ is $\{x \mid x\neq 2, x\neq 3, x\in R\}$.

Use the following information to answer the next part of written-response question 2.

Betty is creating the function $h(x) = \dfrac{(x-a)(x-b)}{(x-c)}$ using the following values for a, b, and c:

$$-3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$$

Betty would like the graph of the function to have a vertical asymptote and two distinct x-intercepts, one of which is negative. She may use the provided values more than once.

b. Provide an example of an equation Betty could use to represent the function, and **determine** the total number of different graphs that would meet Betty's criteria. **[2 marks]**

A possible solution to part b

A possible example could be $h(x) = \dfrac{(x+2)(x-5)}{(x-3)}$.

To determine the total number of different graphs:

The value of a or b must be $-3, -2$, or -1, while the value of the other parameter must then be 0, 1, 2, 3, 4, or 5. This means that there are three options for one factor in the numerator and six options for the other factor in the numerator.

The value of c cannot be the same as the value of a or b; otherwise, the resulting graph would have a point of discontinuity instead of a vertical asymptote. Therefore, there are seven remaining unused options for the factor in the denominator.

Using the fundamental counting principle,

$3 \times 6 \times 7 = 126$

Therefore, 126 different graphs would meet Betty's criteria.

Scoring guide for written-response question 2

PART A

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	The response does not provide any correct information relevant to the comparison of the characteristics of the two functions.
0.5		
1	In the response, the student demonstrates minimal mathematical understanding of the problem by applying an appropriate strategy or some relevant mathematical knowledge to complete initial stages of a solution.	In the response, the student • correctly states which characteristics are the same and which characteristics are different but does not provide supporting details or evidence OR • correctly compares 2 characteristics of both graphs (i.e., the *x*-intercepts and the vertical asymptotes) OR • states 5 correct characteristics of one or both graphs
1.5		
2	In the response, the student demonstrates good mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student • correctly compares 4 characteristics of both graphs OR • correctly states all 10 characteristics for both $f(x)$ and $g(x)$
2.5		
3	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • correctly compares all the characteristics of $f(x)$ and $g(x)$ with supporting details

PART B

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	The response does not contain a valid example or any valid calculations that would lead to determining the number of different graphs.
0.5		
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student • provides a correct example of an equation that meets the criteria OR • provides an example of an equation with one incorrect value and determines the number of possible graphs, but an error is present in the calculation OR • determines the correct number of possible graphs with supporting evidence
1.5		
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • provides a correct example of an equation that meets the criteria and determines the correct number of possible graphs with supporting evidence

Written-response Question 3 Sample Solution

Written Response—5 Marks

3. **a.** **Determine** the value of Angle θ in standard position such that $\csc\theta = -\sqrt{2}$, where $540° \le \theta \le 630°$. **Sketch** and label Angle θ in standard position on the Cartesian plane below. **[2 marks]**

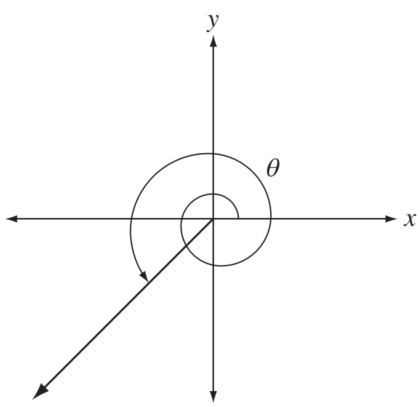
A possible solution to part a

$$\csc\theta = -\sqrt{2}$$

$$\sin\theta = -\frac{1}{\sqrt{2}}$$

The terminal arm is in quadrant III or IV and the reference angle is $45°$.

The angle between $540° \le \theta \le 630°$ with a reference angle of $45°$ is $\theta = 585°$.



Use the following information to answer the next part of written-response question 3.

The terminal arm of Angle β intersects the unit circle at Point $P\left(m, -\frac{\sqrt{3}}{4}\right)$, where $\sec\beta > 0$.

b. Algebraically determine the exact value of $\cos\left(\beta + \frac{\pi}{6}\right)$ in the form $\frac{\sqrt{a}+\sqrt{b}}{c}$, where $a, b, c \in N$. **[3 marks]**

A possible solution to part b

If Point $P\left(m, -\frac{\sqrt{3}}{4}\right)$ intersects the unit circle, then

$$m^2 + \left(-\frac{\sqrt{3}}{4}\right)^2 = 1^2$$

$$m^2 = 1 - \frac{3}{16}$$

$$m^2 = \frac{13}{16}$$

$$m = \pm\frac{\sqrt{13}}{4}$$

Because $\sec\beta > 0$, Angle β is in quadrant IV and $m = \frac{\sqrt{13}}{4}$.

Therefore, Point P is located at $\left(\frac{\sqrt{13}}{4}, -\frac{\sqrt{3}}{4}\right)$ and $\cos\beta = \frac{\sqrt{13}}{4}$ and $\sin\beta = -\frac{\sqrt{3}}{4}$.

So, $\cos\left(\beta + \frac{\pi}{6}\right) = \cos\beta\cos\frac{\pi}{6} - \sin\beta\sin\frac{\pi}{6}$

$$= \left(\frac{\sqrt{13}}{4}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(-\frac{\sqrt{3}}{4}\right)\left(\frac{1}{2}\right)$$

$$= \frac{\sqrt{39}}{8} + \frac{\sqrt{3}}{8}$$

$$\cos\left(\beta + \frac{\pi}{6}\right) = \frac{\sqrt{39}+\sqrt{3}}{8}$$

The exact value of $\cos\left(\beta + \frac{\pi}{6}\right)$ is $\frac{\sqrt{39}+\sqrt{3}}{8}$.

Scoring guide for written-response question 3

PART A

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	The response does not contain any relevant information that would lead to determining Angle θ or a valid sketch of Angle θ.
0.5		
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student • determines a correct coterminal angle for θ, with supporting evidence, within the restriction $0° \le \theta \le 360°$ OR • provides a complete sketch of Angle θ OR • determines a correct reference angle for Angle θ and provides a partially complete sketch
1.5		
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • determines the correct value for Angle θ and provides a complete sketch of the angle

A complete sketch would include the following:

- terminal arm located in quadrant III with a reference angle of approximately 45°
- spiral arrow around the origin indicating the angle contains more than one rotation

PART B

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	The response does not contain any relevant information that would lead to determining the exact value.
0.5		
1	In the response, the student demonstrates minimal mathematical understanding of the problem by applying an appropriate strategy or some relevant mathematical knowledge to complete initial stages of a solution.	In the response, the student • determines the correct possible values of m OR • determines the incorrect value of m and identifies the corresponding value of $\cos\beta$ OR • correctly expands the identity of $\cos\left(\beta + \frac{\pi}{6}\right)$ and identifies the value of $\sin\beta$ OR • determines the value of β and writes the correct decimal approximation of $\cos\left(\beta + \frac{\pi}{6}\right)$
1.5		
2	In the response, the student demonstrates good mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student • determines the correct value of m and identifies the correct values of $\cos\beta$ and $\sin\beta$ OR • determines the incorrect value of m, identifies the correct corresponding values of $\cos\beta$ and $\sin\beta$, and determines the corresponding exact value of $\cos\left(\beta + \frac{\pi}{6}\right)$
2.5		
3	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • determines the correct exact value of $\cos\left(\beta + \frac{\pi}{6}\right)$ with complete supporting evidence

Note: A student that writes the ratios as the angle arguments will receive a maximum score of 2 (i.e., incorrectly writes $\cos\frac{\pi}{6}$ as $\cos\frac{\sqrt{3}}{2}$).