

Written-Response Information

Mathematics 30–2



Alberta Provincial Diploma Examinations

This document was written primarily for:

Students	✓
Teachers	✓ of Mathematics 30–2
Administrators	✓
Parents	
General Audience	
Others	

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Please note that if you cannot access one of the direct website links referred to in this document, you can find diploma examination-related materials on the [Alberta Education website](#).

Introduction

The purpose of this document is to provide examples of written-response questions, sample responses, and scoring rationales as they relate to the scoring guides. This document should be used in conjunction with the [Mathematics 30–2 Program of Studies](#) and the [Mathematics 30–2 Assessment Standards and Exemplars](#) documents, which contain details about the philosophy of the program and the assessment standards. Information about the diploma examination blueprint can be found in the [Mathematics 30-2 Information Bulletin](#). For examples of machine-scored questions, please refer to the [Mathematics 30–2 Released Items](#), which can be found on the [Alberta Education](#) website.

Teachers are encouraged to share the contents of this document with students.

If you have comments or questions regarding this document, please contact Jenny Kim, Mathematics 30–2 Exam Manager, by email at Jenny.Kim@gov.ab.ca or by phone at (780) 415-6127 (dial 310-0000 to be connected toll free).

Intent of the Written-response Component

In 2016, it was announced that high school mathematics diploma examinations will integrate a written-response component that will require students to communicate their understanding of mathematical concepts and demonstrate their algebraic skills. Therefore, the written-response component is designed to complement the machine-scored portion of the diploma examination by allowing for greater coverage of the learning outcomes in the program of studies.

The written-response component also provides an opportunity to address the mathematical processes outlined in the *Mathematics 30–2 Program of Studies*. Of the seven mathematical processes, the written-response component will focus primarily on communication (C), problem solving (PS), connections (CN), reasoning (R), and visualization (V).

Each specific outcome in the *Mathematics 30–2 Program of Studies* lists the related mathematical processes for that outcome. If technology (T) is not listed as a process, students are expected to meet the outcome **without** the use of technology and must use an algebraic process to receive credit on a question involving the outcome.

Written-response Question Design

The written-response component is designed to assess the degree to which students can draw on their mathematical experiences to solve problems, explain mathematical concepts, and demonstrate their algebraic skills. A written-response question will cover more than one specific outcome and will require students to make connections between concepts. Each written-response question will consist of four parts and will address multiple cognitive levels. Students should be encouraged to try to solve the problems in all parts, as an attempt at a solution may be worth partial marks.

Students may be asked to solve, explain, or prove in a written-response question. Students are required to know the definitions and expectations of directing words such as **algebraically**, **compare**, **determine**, **evaluate**, **justify**, and **sketch**. A list of these directing words and their definitions can be found on the Alberta Education website.

General Scoring Guides

The General Scoring Guides, developed in consultation with teachers and Alberta Education staff, describe the criteria and performance level at each score-point value. These General Scoring Guides will be used to develop specific scoring descriptions for each written-response question.

In scoring the written-response questions, markers will evaluate how well students

- demonstrate their understanding of the problem or the mathematical concept;
- correctly apply mathematical knowledge and skills;
- use problem-solving strategies and explain their solutions and procedures;
- communicate their solutions and mathematical ideas.

1-mark Part

Score	General Scoring Guide
NR	No response is provided.
0	In the response, the student does not address the question or provides a solution that is invalid.
0.5	
1	In the response, the student applies appropriate mathematical knowledge to find a complete and correct solution.

2-mark Part

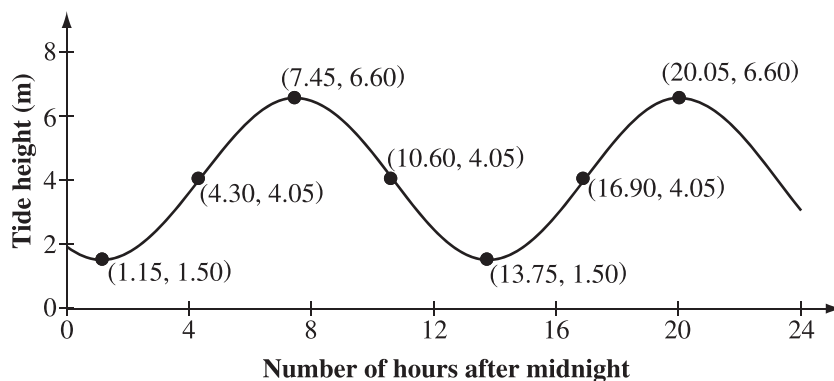
Score	General Scoring Guide
NR	No response is provided.
0	In the response, the student does not address the question or provides a solution that is invalid.
0.5	
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.
1.5	
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.

Specific Scoring Guides for each written-response question will provide detailed descriptions to clarify expectations of student performance at each benchmark score of 0, 1, and 2. A student response that does not meet the performance level of a benchmark score may receive an augmented score of 0.5 or 1.5. Descriptions of these augmented scores will be determined with teachers at each marking session and are not an exhaustive list. Each part will be scored separately and the scores will be combined for a total of 7 marks. Each question will begin with a 1-mark part followed by three 2-mark parts.

Written-response Question 1

Use the following information to answer written-response question 1.

The English Channel is a body of water that separates England and France, and each year swimmers from around the world attempt to swim across it. In preparation for her solo swim, Leah has researched the predicted tide height for the particular day that she plans to swim. The predicted heights of the tide, in metres, as a function of the number of hours after midnight, are shown in the graph below.



Written Response—7 Marks

1. a. • State the sinusoidal regression function that represents the graph above in the form $y = a \sin(bx - 2.14) + d$, where y represents the height of the tide, in metres, x hours after midnight. If necessary, round values to the nearest hundredth.

A POSSIBLE SOLUTION to part a, bullet 1

$$y = 2.55 \sin(0.50x - 2.14) + 4.05$$

- Leah must start her solo swim at a falling tide of 5 m in height. Calculate the fewest number of hours after midnight when Leah should start her swim, to the nearest hundredth of an hour. Mark and label this point on the graph above.

A POSSIBLE SOLUTION to part a, bullet 2

$$y = 2.55\sin(0.50x - 2.14) + 4.05$$

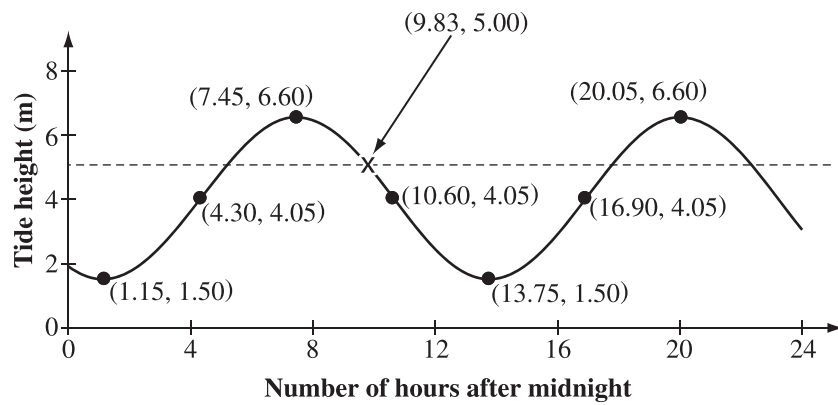
$$5 = 2.55\sin(0.50x - 2.14) + 4.05$$

$$x = 9.834\ 448\ 2\dots$$

$$x = 9.83 \text{ hours after midnight}$$

OR $x = 9.80$ hours after midnight

(if using the regression function with rounded values)



Use the following information to answer the next part of the question.

Data for the number of male and female solo swimmers crossing the English Channel were collected and recorded annually from 1954 to 2016.

The number of male solo swimmers, M , and the number of female solo swimmers, F , can be modelled by the exponential functions shown below, where x represents the number of years after 1954.

$$M = 6(1.038)^x$$

$$F = 4(1.047)^x$$

- b. • Using the numerical values in the functions above, **compare** the information that these values provide about the number of male solo swimmers and the number of female solo swimmers.

A POSSIBLE SOLUTION to part b, bullet 1

The coefficients 6 and 4 represent the number of male and female solo swimmers, respectively, crossing the English Channel in 1954 (Year 0); i.e., there were 2 more male solo swimmers crossing the English Channel in 1954 than female solo swimmers.

The bases 1.038 and 1.047, both being values greater than 1, indicate that the numbers of male and female solo swimmers are increasing exponentially. The number of female solo swimmers increases at an average rate of 4.7% per year, which is faster than the average increase of 3.8% per year for male solo swimmers.

- **Algebraically determine** the year in which the number of female solo swimmers to cross the English Channel will reach 180.

A POSSIBLE SOLUTION to part b, bullet 2

$$F = 4(1.047)^x$$

$$180 = 4(1.047)^x$$

$$45 = (1.047)^x$$

$$\log_{1.047}(45) = x$$

$$82.881\ 581\ 03 = x$$

It is expected that 180 female solo swimmers will cross the English channel 83 years after 1954 or in 2037.

Specific Scoring Guide for Written-response Question 1

Part a, bullet 1:

Score	General Scoring Guide	Specific Scoring Guide
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	In the response, the student <ul style="list-style-type: none"> • writes an incorrect sinusoidal function.
0.5		<i>For example, the student could</i> <ul style="list-style-type: none"> • <i>correctly state the parameter values, but not express as a function</i> <i>OR</i> <ul style="list-style-type: none"> • <i>correctly state 2 out of the 3 parameters, and express as a function.</i>
1	In the response, the student applies appropriate mathematical knowledge to find a complete and correct solution.	In the response, the student <ul style="list-style-type: none"> • writes a correct sinusoidal regression function.
Notes: Rounding is a minor error and a student can still receive full marks.		

Please note that the augmented score descriptions (i.e., the italicized statements) are determined at marking sessions. They are not an exhaustive list.

Part a, bullet 2:

Score	General Scoring Guide	Specific Scoring Guide
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	In the response, the student <ul style="list-style-type: none"> marks and labels the point at the first rising tide, and then estimates the time from the graph.
0.5		<i>For example, the student could</i> <ul style="list-style-type: none"> mark and label the point at the first rising tide, and correctly calculate the time at this point OR <ul style="list-style-type: none"> calculate the correct number of hours after midnight at the second falling tide, but not mark and label the point on the graph.
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student <ul style="list-style-type: none"> marks and labels the correct point, and then estimates the time from the graph OR <ul style="list-style-type: none"> calculates the correct number of hours after midnight, but does not mark and label the correct point on the graph.
1.5		<i>For example, the student could</i> <ul style="list-style-type: none"> mark and label the point at the second falling tide, and correctly calculate the time at this point.
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student <ul style="list-style-type: none"> calculates the correct number of hours after midnight, and marks and labels the correct point on the graph.
<p>Notes:</p> <ul style="list-style-type: none"> • Rounding is a minor error and a student can still receive full marks. • Students can still receive full marks if they use an incorrect function from bullet 1 to correctly solve this bullet. 		

Part b, bullet 1:

Score	General Scoring Guide	Specific Scoring Guide
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	In the response, the student <ul style="list-style-type: none"> incorrectly describes the parameter values with no comparison and no reference to context.
0.5		<i>For example, the student could</i> <ul style="list-style-type: none"> <i>provide a definition of a and b with no reference to numerical values or context.</i>
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student <ul style="list-style-type: none"> correctly compares the characteristics represented by one parameter value within the context OR <ul style="list-style-type: none"> correctly describes both parameter values with no reference to the context OR <ul style="list-style-type: none"> correctly describes both parameter values with no comparison provided.
1.5		<i>For example, the student could</i> <ul style="list-style-type: none"> <i>correctly describe both parameter values but then provide an incomplete comparison of the characteristics.</i>
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student <ul style="list-style-type: none"> correctly compares the characteristics represented by the parameter values within the context.

Part b, bullet 2:

Score	General Scoring Guide	Specific Scoring Guide
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	In the response, the student <ul style="list-style-type: none"> substitutes 180 in for x OR <ul style="list-style-type: none"> states the correct answer, with no supporting work.
0.5		<i>For example, the student could</i> <ul style="list-style-type: none"> <i>correctly substitute 180 in for F and find the graphical solution, expressed as an actual calendar year or years after 1954.</i>
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student <ul style="list-style-type: none"> correctly substitutes 180 in for F, but makes an error such as <ul style="list-style-type: none"> incorrectly converting to logarithmic form (e.g., $\log_{45}1.047$) or <ul style="list-style-type: none"> taking the log of both sides of the equation, but incorrectly isolating x OR <ul style="list-style-type: none"> incorrectly multiplies 4 by 1.047, but nevertheless gets the answer 4 (1958).
1.5		<i>For example, the student could</i> <ul style="list-style-type: none"> <i>correctly determine the number of years algebraically, but round the final answer incorrectly, or fail to express it as an actual calendar year or years after 1954</i> OR <ul style="list-style-type: none"> <i>correctly show all algebraic work, but obtain an incorrect solution.</i>
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student <ul style="list-style-type: none"> correctly determines the number of years algebraically, expressed as an actual calendar year or years after 1954.

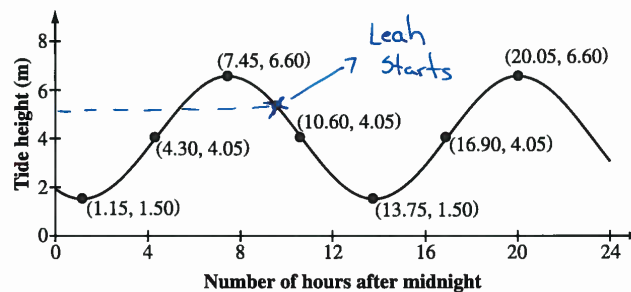
Sample Responses to Written-response Question 1

Note: The sample responses are intended to inform teachers and students of how the scoring guide is applied to specific questions and to illustrate the expectations for student performance.

Sample Response 1

Use the following information to answer written-response question 1.

The English Channel is a body of water that separates England and France, and each year swimmers from around the world attempt to swim across it. In preparation for her solo swim, Leah has researched the predicted tide height for the particular day that she plans to swim. The predicted heights of the tide, in metres, as a function of the number of hours after midnight, are shown in the graph below.



Written Response—7 Marks

1. a. • State the sinusoidal regression function that represents the graph above in the form $y = a \sin(bx - 2.14) + d$, where y represents the height of the tide, in metres, x hours after midnight. If necessary, round values to the nearest hundredth.

$$y = 2.61 \sin(0.52x - 2.07) + 4.10$$

- Leah must start her solo swim at a falling tide of 5 m in height. Calculate the fewest number of hours after midnight when Leah should start her swim, to the nearest hundredth of an hour. Mark and label this point on the graph above.

$$5 = 2.61 \sin(0.52x - 2.07) + 4.10$$

$$x = 9.4 \text{ hours}$$

Use the following information to answer the next part of the question.

Data for the number of male and female solo swimmers crossing the English Channel were collected and recorded annually from 1954 to 2016.

The number of male solo swimmers, M , and the number of female solo swimmers, F , can be modelled by the exponential functions shown below, where x represents the number of years after 1954.

$$M = 6(1.038)^x$$

$$F = 4(1.047)^x$$

- b. • Using the numerical values in the functions above, **compare** the information that these values provide about the number of male solo swimmers and the number of female solo swimmers.

1954 → 2016

↓
0

$M = 6(1.038)^x \rightarrow$ no. of years

↑
no. of male swimmers that is increasing by 3.8% increase

of swimmers at start of 1954

$f = 4(1.047)^x \rightarrow$ no. of yrs

↑
of swimmers @ start of 1954

↓
of female swimmers that is increasing 4.7% increase

There are more male swimmers to start with (6-4) however the amount of female swimmers increases more than male (4.7-3.8)

- **Algebraically determine** the year in which the number of female solo swimmers to cross the English Channel will reach 180.

$$f = 4(1.047)^x$$

$$180 = 4(1.047)^x$$

$$45 = 1.047^x$$

$$x = \log_{45} 1.047$$

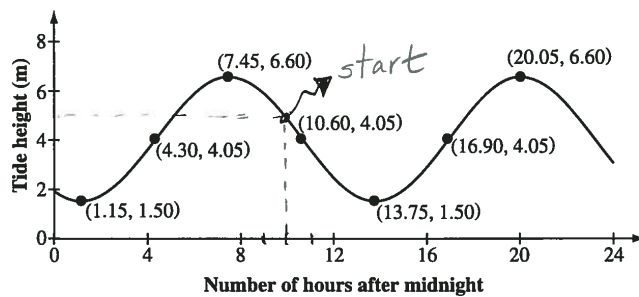
$$x = 0.01 \text{ yrs } ??$$

Total Score-5	Rationale
Part a Bullet 1: 0 Bullet 2: 2	In part a, bullet 1, the sinusoidal regression function is incorrect. However, the correct calculation process is used in bullet 2, with an appropriate point marked and labelled on the graph.
Part b Bullet 1: 2 Bullet 2: 1	In part b, bullet 1 is correct. The response in bullet 2 starts with a correct algebraic process, but an error occurred in the conversion of exponential form to logarithmic form.

Sample Response 2

Use the following information to answer written-response question 1.

The English Channel is a body of water that separates England and France, and each year swimmers from around the world attempt to swim across it. In preparation for her solo swim, Leah has researched the predicted tide height for the particular day that she plans to swim. The predicted heights of the tide, in metres, as a function of the number of hours after midnight, are shown in the graph below.



Written Response—7 Marks

1. a. • State the sinusoidal regression function that represents the graph above in the form $y = a \sin(bx - 2.14) + d$, where y represents the height of the tide, in metres, x hours after midnight. If necessary, round values to the nearest hundredth.

$$y = 2.55 \sin(0.50x - 2.14) + 4.05$$

- Leah must start her solo swim at a falling tide of 5 m in height. Calculate the fewest number of hours after midnight when Leah should start her swim, to the nearest hundredth of an hour. Mark and label this point on the graph above.

10 hours after midnight.

Use the following information to answer the next part of the question.

Data for the number of male and female solo swimmers crossing the English Channel were collected and recorded annually from 1954 to 2016.

The number of male solo swimmers, M , and the number of female solo swimmers, F , can be modelled by the exponential functions shown below, where x represents the number of years after 1954.

$$M = 6(1.038)^x$$

$$F = 4(1.047)^x$$

- b. • Using the numerical values in the functions above, **compare** the information that these values provide about the number of male solo swimmers and the number of female solo swimmers.

Number of males are 6 and number of girls are 4.
The number of girl swimmer grow 4.7% and the guy swimmers grow slower at 3.8%

- **Algebraically determine** the year in which the number of female solo swimmers to cross the English Channel will reach 180.

$$F = 4(1.047)^x$$

$$180 = 4(1.047)^x$$

$$45 = (1.047)^x$$

$$x = \log_{1.047} 45$$

$$x = 82.8$$

83 yrs

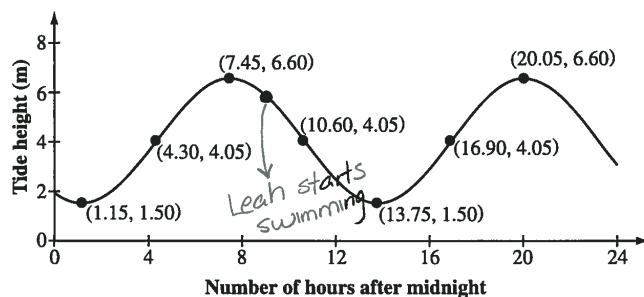
2037

Total Score–5.5	Rationale
Part a Bullet 1: 1 Bullet 2: 1	In part a, bullet 1 is correct. In bullet 2, the correct point is marked and labelled, but the time was estimated from the graph.
Part b Bullet 1: 1.5 Bullet 2: 2	In part b, bullet 1, the response reflects a good understanding of the growth rate for the number of male and female solo swimmers. The number of male and female solo swimmers is stated but then no comparison is made. Bullet 2 is correct.

Sample Response 3

Use the following information to answer written-response question 1.

The English Channel is a body of water that separates England and France, and each year swimmers from around the world attempt to swim across it. In preparation for her solo swim, Leah has researched the predicted tide height for the particular day that she plans to swim. The predicted heights of the tide, in metres, as a function of the number of hours after midnight, are shown in the graph below.



Written Response—7 Marks

1. a. • State the sinusoidal regression function that represents the graph above in the form $y = a \sin(bx - 2.14) + d$, where y represents the height of the tide, in metres, x hours after midnight. If necessary, round values to the nearest hundredth.

$$2.55 \sin(0.499x - 2.14) + 4.05$$

- Leah must start her solo swim at a falling tide of 5 m in height. Calculate the fewest number of hours after midnight when Leah should start her swim, to the nearest hundredth of an hour. Mark and label this point on the graph above.

$$y_1 = 2.55 \sin(0.499x - 2.14) + 4.05$$

$$y_2 = 5 \quad (\text{found intersection})$$

start swim 9.8 hours after midnight

Use the following information to answer the next part of the question.

Data for the number of male and female solo swimmers crossing the English Channel were collected and recorded annually from 1954 to 2016.

The number of male solo swimmers, M , and the number of female solo swimmers, F , can be modelled by the exponential functions shown below, where x represents the number of years after 1954.

$$M = 6(1.038)^x$$

$$F = 4(1.047)^x$$

- b. • Using the numerical values in the functions above, **compare** the information that these values provide about the number of male solo swimmers and the number of female solo swimmers.

$$M = 6(1.038)^x$$

↑
number of
male in first
year

$$F = 4(1.047)^x$$

↑
number of
females in the
first year

- **Algebraically determine** the year in which 180 female solo swimmers will cross the English Channel.

$$180 = 4(1.047)^x$$

$$(82.9, 180)$$

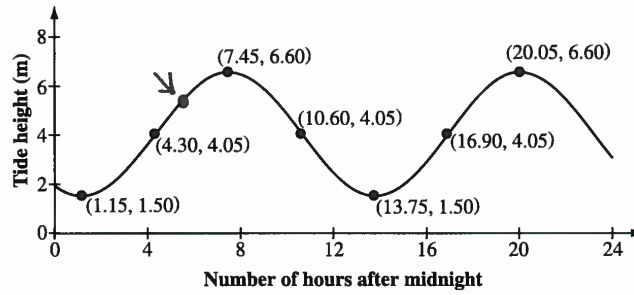
In 83 years.

Total Score–4	Rationale
Part a Bullet 1: 0.5 Bullet 2: 2	In part a, bullet 1, while the correct parameter values are stated, the answer is written as an expression and not as a function. Bullet 2 is correct.
Part b Bullet 1: 1 Bullet 2: 0.5	In part b, bullet 1, only one parameter is correctly identified and compared. The response in bullet 2 shows the correct substitution of 180 into the function. However, the directing word indicates that an algebraic solution is required.

Sample Response 4

Use the following information to answer written-response question 1.

The English Channel is a body of water that separates England and France, and each year swimmers from around the world attempt to swim across it. In preparation for her solo swim, Leah has researched the predicted tide height for the particular day that she plans to swim. The predicted heights of the tide, in metres, as a function of the number of hours after midnight, are shown in the graph below.



Written Response—7 Marks

1. a. • State the sinusoidal regression function that represents the graph above in the form $y = a \sin(bx - 2.14) + d$, where y represents the height of the tide, in metres, x hours after midnight. If necessary, round values to the nearest hundredth.

$$y = 2.55 \sin(0.499x - 2.14) + 4.05$$

- Leah must start her solo swim at a falling tide of 5 m in height. Calculate the fewest number of hours after midnight when Leah should start her swim, to the nearest hundredth of an hour. Mark and label this point on the graph above.

She should start swimming
at 5.04 hours.

Use the following information to answer the next part of the question.

Data for the number of male and female solo swimmers crossing the English Channel were collected and recorded annually from 1954 to 2016.

The number of male solo swimmers, M , and the number of female solo swimmers, F , can be modelled by the exponential functions shown below, where x represents the number of years after 1954.

$$M = 6(1.038)^x$$

$$F = 4(1.047)^x$$

- b. • Using the numerical values in the functions above, **compare** the information that these values provide about the number of male solo swimmers and the number of female solo swimmers.

As exp. function $y = ab^x$
 a is the starting amount
 b is the rate of increase or decrease

- **Algebraically determine** the year in which 180 female solo swimmers will cross the English Channel.

$$180 = 4(1.047)^x$$

$$45 = 1.047^x$$

$$x = \log_{1.047} 45$$

$$x = 82$$

Year 2036

Total Score–3.5	Rationale
Part a Bullet 1: 1 Bullet 2: 0.5	In part a, bullet 1, although there is a minor rounding error in parameter b , this did not hinder the understanding of the response. In bullet 2, the first rising tide is marked on the graph, which indicates a lack of mathematical understanding of the problem.
Part b Bullet 1: 0.5 Bullet 2: 1.5	In part b, bullet 1, the response lacks any reference to the numerical values or the context in the problem. The response in bullet 2 contains an error in rounding which does hinder the mathematical understanding of the context. In 2036, the number of female solo swimmers to cross the English Channel will not reach 180.

Written-response Question 2

Use the following information to answer written-response question 2.

In a Math 30–2 class, the teacher gives the class the following rational expressions.

$$\frac{3x^2 + 18x}{x^2 - 36} \qquad \frac{28}{4x - 24}$$

Written Response—7 Marks

2. a. • State all the non-permissible values of x in the expressions above.

A POSSIBLE SOLUTION to part a, bullet 1

Since $x^2 - 36 = (x - 6)(x + 6)$ and $4x - 24 = 4(x - 6)$, the non-permissible values of x are -6 and 6 .

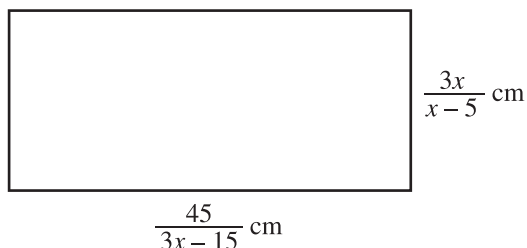
- **Determine** the simplified product of the two expressions above.

A POSSIBLE SOLUTION to part a, bullet 2

$$\begin{aligned} & \frac{3x^2 + 18x}{x^2 - 36} \cdot \frac{28}{4x - 24} \\ &= \frac{3x\cancel{(x+6)}}{(x-6)\cancel{(x+6)}} \cdot \frac{28}{4(x-6)} \\ &= \frac{84x}{4(x-6)(x-6)} \\ &= \frac{21x}{(x-6)^2} \end{aligned}$$

Use the following additional information to answer the next part of the question.

The Math 30–2 teacher also gives the class a diagram of a rectangle with dimensions represented by rational expressions, where $x > 5$, as shown below. She reminds the class that the perimeter of a rectangle is the sum of the lengths of all the sides.



- b. • Given that the perimeter of the rectangle above is 46 cm, write an equation that can be used to solve for x . **Algebraically determine** the value of x , to the nearest tenth, using this equation.

A POSSIBLE SOLUTION to part b, bullet 1

$$46 = 2\left(\frac{45}{3x-15}\right) + 2\left(\frac{3x}{x-5}\right)$$

$$23 = \frac{45}{3(x-5)} + \frac{3x}{x-5}$$

$$23 = \frac{15}{x-5} + \frac{3x}{x-5}$$

$$23(x-5) = 15 + 3x$$

$$23x - 115 = 15 + 3x$$

$$20x = 130$$

$$x = 6.5$$

- **Explain** why $x > 5$ for the rectangle above.

A POSSIBLE SOLUTION to part b, bullet 2

The expressions that represent the length and width of the rectangle both contain the factor $(x - 5)$ in the denominator. Therefore, x must be greater than 5 in the expression since the side lengths must always be positive values.

Specific Scoring Guide for Written-response Question 2

Part a, bullet 1:

Score	General Scoring Guide	Specific Scoring Guide
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	In the response, the student <ul style="list-style-type: none"> • states incorrect non-permissible values.
0.5		<i>For example, the student could</i> <ul style="list-style-type: none"> • <i>state only one of the correct non-permissible values of x</i> <i>OR</i> <ul style="list-style-type: none"> • <i>state all the correct non-permissible values of x, and include 0.</i>
1	In the response, the student applies appropriate mathematical knowledge to find a complete and correct solution.	In the response, the student <ul style="list-style-type: none"> • states all the correct non-permissible values of x.

Part a, bullet 2:

Score	General Scoring Guide	Specific Scoring Guide
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	In the response, the student <ul style="list-style-type: none"> shows the multiplication statement with no factoring.
0.5		<i>For example, the student could</i> <ul style="list-style-type: none"> <i>correctly factor two of the polynomials, but simplify incorrectly</i> OR <ul style="list-style-type: none"> <i>multiply the expressions correctly with no factoring.</i>
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student <ul style="list-style-type: none"> correctly factors at least two of the polynomials, but simplifies incompletely with common factors remaining OR <ul style="list-style-type: none"> correctly factors all the polynomials and reduces, but makes an error by incorrectly finding the sum.
1.5		<i>For example, the student could</i> <ul style="list-style-type: none"> <i>correctly determine the product, but not reduce the coefficients</i> OR <ul style="list-style-type: none"> <i>correctly factor all the polynomials and reduce, but make a minor error in the simplified product.</i>
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student <ul style="list-style-type: none"> correctly determines the simplified product.
<p>Notes: Students do not need to restate the non-permissible values in bullet 2.</p>		

Part b, bullet 1:

Score	General Scoring Guide	Specific Scoring Guide
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	In the response, the student <ul style="list-style-type: none"> • writes an incorrect equation.
0.5		<i>For example, the student could</i> <ul style="list-style-type: none"> • <i>write a correct equation but solve incorrectly.</i>
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student <ul style="list-style-type: none"> • writes $L + W = 46$ and solves for x in this equation correctly (i.e., $x = 13$) OR <ul style="list-style-type: none"> • writes a correct equation and eliminates all the fractions correctly, but obtains an incorrect linear equation.
1.5		<i>For example, the student could</i> <ul style="list-style-type: none"> • <i>write a correct equation and eliminate all the fractions correctly, but make a minor error in solving the linear equation.</i>
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student <ul style="list-style-type: none"> • writes a correct equation and determines the correct solution for x.

Note: Although the solution to this question is an explanation of why x must be greater than 5, some responses had explanations of why x cannot equal 5 and/or why x cannot be less than 5, which is reflected in the specific scoring guide below.

Part b, bullet 2:

Score	General Scoring Guide	Specific Scoring Guide
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	In the response, the student <ul style="list-style-type: none"> provides an incorrect explanation OR <ul style="list-style-type: none"> states that $x \neq 5$.
0.5		<i>For example, the student could</i> <ul style="list-style-type: none"> state when $x < 5$ and $x = 5$ result in zero and negative values, respectively, with no reference to the side lengths OR <ul style="list-style-type: none"> explain why $x = 5$ is a non-permissible value with no reference to the side lengths.
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student <ul style="list-style-type: none"> clearly explains why x cannot be less than 5, with reference to the side lengths OR <ul style="list-style-type: none"> clearly explains why $x \neq 5$, with reference to the side lengths.
1.5		<i>For example, the student could</i> <ul style="list-style-type: none"> provide an incomplete explanation of why x must be greater than 5 for the side lengths.
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student <ul style="list-style-type: none"> clearly explains, referencing the side lengths, why x must be greater than 5 OR <ul style="list-style-type: none"> clearly explains, referencing the side lengths, why x cannot be less than 5 and why x cannot equal 5.
Note: Students must make reference to the side lengths of the rectangle in order to receive full marks.		

Sample Responses to Written-response Question 2

Sample Response 1

Use the following information to answer written-response question 2.

In a Math 30–2 class, the teacher gives the class the following rational expressions.

$$\frac{3x^2 + 18x}{x^2 - 36} \qquad \frac{28}{4x - 24}$$

Written Response—7 Marks

2. a. • State all the non-permissible values of x in the expressions above.

$$x \neq 6, -6$$

- Determine the simplified product of the two expressions above.

$$\frac{-3x^2 + 18x}{x^2 - 36} \times \frac{28}{4x - 24}$$

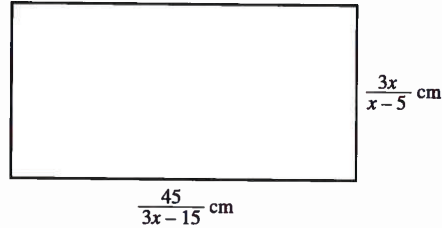
$$\frac{3x(x+6)}{(x+6)(x-6)} \times \frac{28}{4(x-6)}$$

$$= \frac{84x}{4(x-6)^2} \quad x \neq 6, -6$$

$$= \frac{x}{21(x-6)^2}$$

Use the following additional information to answer the next part of the question.

The Math 30–2 teacher also gives the class a diagram of a rectangle with dimensions represented by rational expressions, where $x > 5$, as shown below. She reminds the class that the perimeter of a rectangle is the sum of the lengths of all the sides.



- b. • Given that the perimeter of the rectangle above is 46 cm, write an equation that can be used to solve for x . **Algebraically determine** the value of x , to the nearest tenth, using this equation.

$$2\left(\frac{45}{3x-15}\right) + 2\left(\frac{3x}{x-5}\right) = 46$$

$$2\left(\frac{45}{3(x-5)}\right) + 2\left(\frac{3x}{x-5}\right) = 46$$

$$\left[\frac{90}{6(x-5)}\right]^{6(x-5)} + \left[\frac{6x}{x-5}\right]^{6(x-5)} = 46$$

$$90 + 36x = 46(6(x-5))$$

$$\begin{array}{r} 90 \\ + 1380 \\ \hline 1470 \\ \underline{240} \end{array} \quad \begin{array}{r} + 36x = 276x - 1380 \\ - 36x \\ \hline = 240x \\ \\ x = 6.125 \end{array} \quad \begin{array}{r} - 1380 \\ + 1380 \\ \hline = 276(x-5) \\ 276x - 1380 \end{array}$$

- Explain why $x > 5$ for the rectangle above.

eg: $\frac{45}{3(x) - 15}$

$x > 5$

$\frac{3x}{(x) - 5}$

If x is equal to 5, the denominators would be 0, and if it's less than 5, it will be negative

Total Score-4	Rationale
Part a Bullet 1: 1 Bullet 2: 1.5 Part b Bullet 1: 1 Bullet 2: 0.5	<p>In part a, bullet 1 is correct. In bullet 2, a minor error is made in the simplification of the coefficients and 21 is mistakenly written in the denominator.</p> <p>In part b, bullet 1, the equation is correct but an error is made when each side length is multiplied by 2. Although an incorrect solution is obtained, appropriate mathematical knowledge was applied to eliminate all the fractions correctly. For bullet 2, the response states the value of the denominators of the expression when $x < 5$ and when $x = 5$, but it does not reflect a complete understanding of the restriction within the given context.</p>

Sample Response 2

Use the following information to answer written-response question 2.

In a Math 30–2 class, the teacher gives the class the following rational expressions.

$$\frac{3x^2 + 18x}{x^2 - 36} \qquad \frac{28}{4x - 24}$$

Written Response—7 Marks

2. a. • State all the non-permissible values of x in the expressions above.

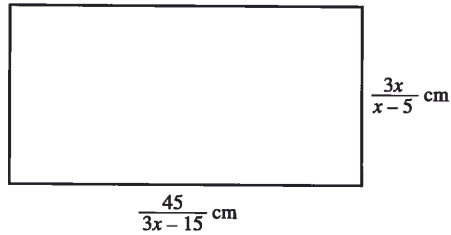
$$x \neq 36, 24$$

- Determine the simplified product of the two expressions above.

$$\begin{aligned} &= \left(\frac{3x^2 + 18x}{x^2 - 36} \right) \left(\frac{28}{4x - 24} \right) \\ &= \frac{84x^2 + 504x}{4x^3 - 24x^2 - 144x + 864} \\ &= \frac{21x^2 + 126x}{x^3 - 6x^2 - 36x + 216} \end{aligned}$$

Use the following additional information to answer the next part of the question.

The Math 30–2 teacher also gives the class a diagram of a rectangle with dimensions represented by rational expressions, where $x > 5$, as shown below. She reminds the class that the perimeter of a rectangle is the sum of the lengths of all the sides.



- b. • Given that the perimeter of the rectangle above is 46 cm, write an equation that can be used to solve for x . **Algebraically determine** the value of x , to the nearest tenth, using this equation.

$$\frac{45}{3x-15} * \frac{3x}{x+5} = 23$$

$$\frac{45}{-15} * \frac{1}{x+5} = 23$$

$$45(x+5) + (-15) = 23$$

$$45(x+5) = 38$$

$$45x + 225 = 38$$

$$\frac{45x}{45} = \frac{-187}{45}$$

$$? x = -4.15$$

~~$$3x(3x-15) \rightarrow 9x^2 - 45x$$

$$9x^2 - 45x + 45x + 225$$~~

- Explain why $x > 5$ for the rectangle above.

because ~~if~~ if x was less than 5 than the side length would negative # which is impossible for a side length

Total Score–2	Rationale
Part a Bullet 1: 0 Bullet 2: 0.5	In part a, bullet 1, the incorrect non-permissible values are stated. In bullet 2, a correct product is determined, but it is not simplified completely.
Part b Bullet 1: 0.5 Bullet 2: 1	In part b, bullet 1, a correct equation is written. However, incorrect algebraic procedures are then used to find the solution. In the response for bullet 2, a complete explanation of why x cannot be less than 5 is provided, but no explanation is given of why $x \neq 5$.

Sample Response 3

Use the following information to answer written-response question 2.

In a Math 30–2 class, the teacher gives the class the following rational expressions.

$$\frac{3x^2 + 18x}{x^2 - 36} \qquad \frac{28}{4x - 24}$$

Written Response—7 Marks

2. a. • State all the non-permissible values of x in the expressions above.

$$x \neq 6, -6, 0$$

- Determine the simplified product of the two expressions above.

$$\begin{array}{l} \frac{28}{4x-24} \cdot \frac{3x^2+18x}{x^2-36} \\ \frac{28}{4(x-6)} \cdot \frac{6x(x+6)}{(x+6)(x-6)} \end{array}$$

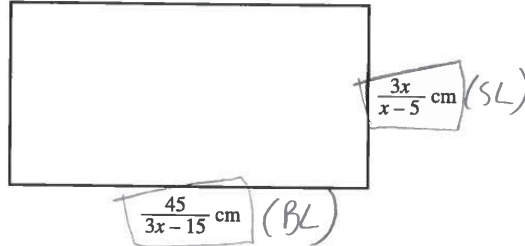
$$\frac{28}{4(x-6)} \cdot \frac{6x(x+6)}{(x+6)(x-6)}$$

simplified

$$7.6x = 42x$$

Use the following additional information to answer the next part of the question.

The Math 30–2 teacher also gives the class a diagram of a rectangle with dimensions represented by rational expressions, where $x > 5$, as shown below. She reminds the class that the perimeter of a rectangle is the sum of the lengths of all the sides.



- b. • Given that the perimeter of the rectangle above is 46 cm, write an equation that can be used to solve for x . **Algebraically determine** the value of x , to the nearest tenth, using this equation.

$$P = [(BL) + (SL)] \times 2$$

$$\frac{15}{x-5} + \frac{15}{x-5} + \frac{3x}{x-5} + \frac{3x}{x-5} = \frac{30+6x}{(x-5)}$$

$$\frac{45}{3x-15} \rightarrow \frac{15}{x-5}$$

$$\frac{6x+30}{x-5} = 46$$

$$\frac{6x+30}{x-5}$$

$$6x+30 = 46(x-5)$$

$$\begin{array}{r} 6x+30 \\ +30 \end{array} = \begin{array}{r} 46x-230 \\ +30 \end{array}$$

$$\frac{6(x+5)}{x-5} = 46$$

$$\begin{array}{r} 6x \\ -6x \end{array} = \begin{array}{r} 46x-200 \\ -6x \end{array}$$

$$0 = 40x - 200$$

$$\frac{200}{40} = \frac{40x}{40} \quad \boxed{x=5}$$

- **Explain** why $x > 5$ for the rectangle above.

$x \neq 5$ because you cannot divide by 0

Total Score–3.5	Rationale
Part a Bullet 1: 0.5 Bullet 2: 1	In part a, bullet 1, all the factors containing x in the numerators and denominators are incorrectly used to identify the restrictions. In bullet 2, the polynomials are factored correctly, but the rational expressions are neither multiplied nor simplified completely.
Part b Bullet 1: 1.5 Bullet 2: 0.5	In part b, bullet 1, the equation written and the algebraic process used to eliminate all the fractions are correct. However, a minor error was made in isolating the variable in the simplified linear equation. In the response to bullet 2, there was no reference to the side lengths and no explanation was provided for why x cannot be less than 5.

Sample Response 4

Use the following information to answer written-response question 2.

In a Math 30–2 class, the teacher gives the class the following rational expressions.

$$\frac{3x^2 + 18x}{x^2 - 36} \qquad \frac{28}{4x - 24}$$

Written Response—7 Marks

2. a. • State all the non-permissible values of x in the expressions above.

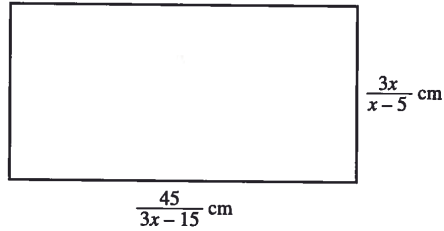
$$x \neq \pm 6$$

- Determine the simplified product of the two expressions above.

$$\begin{aligned} & \frac{28}{4x-24} \times \frac{3x^2+18x}{x^2-36} \\ = & \frac{28}{4(x-6)} \times \frac{3x(\cancel{x+6})}{(x-6)(\cancel{x+6})} \\ = & \frac{28}{4(x-6)} \times \frac{3x}{(x-6)} \\ = & \frac{84x}{4(x-6)^2} \end{aligned}$$

Use the following additional information to answer the next part of the question.

The Math 30–2 teacher also gives the class a diagram of a rectangle with dimensions represented by rational expressions, where $x > 5$, as shown below. She reminds the class that the perimeter of a rectangle is the sum of the lengths of all the sides.



$$\left(\frac{15}{x-5}\right)$$

- b. • Given that the perimeter of the rectangle above is 46 cm, write an equation that can be used to solve for x . **Algebraically determine** the value of x , to the nearest tenth, using this equation.

$$\frac{45}{3x-15} + \frac{45}{3(x-5)} + \frac{3x}{x-5} + \frac{3x}{x-5} = 46 \text{ cm}$$

$$\frac{30}{x-5} + \frac{6x}{x-5} = 46 \text{ cm}$$

$$\left(\frac{6(x+5)}{(x-5)} = 46 \text{ cm}\right)(x-5)$$

$$6x + 5 = 46x - 230$$

$$-6x + 230 = -6x + 230$$

$$\frac{235}{40} = \frac{40x}{40}$$

$$x = 5.875 \rightarrow \boxed{x = 5.9}$$

- **Explain** why $x > 5$ for the rectangle above.

If x is equal to 5, the denominators will equal to zero. The numbers cannot be divided by zero. If x is less than 5, the answer would be negative. There is no such thing as a negative centimeter.

Total Score-6	Rationale
Part a Bullet 1: 1 Bullet 2: 1.5	In part a, bullet 1 is correct. In bullet 2, the product is not simplified completely since the coefficients can still be reduced to lowest terms.
Part b Bullet 1: 1.5 Bullet 2: 2	In part b, bullet 1, a minor transposing error was made when the simplified linear equation was solved. The response in bullet 2 is complete.

Explanation of Cognitive Levels

Procedural

The assessment of students' knowledge of mathematical procedures should involve recognition, execution, and verification of appropriate procedures and the steps contained within them. The use of technology can allow for conceptual understanding prior to specific skill development or vice versa. Students must appreciate that procedures are created or generated to meet specific needs in an efficient manner and thus can be modified or extended to fit new situations. Assessment of students' procedural knowledge will not be limited to an evaluation of their proficiency in performing procedures, but will be extended to reflect the skills presented above.

Conceptual

An understanding of mathematical concepts goes beyond a mere recall of definitions and recognition of common examples. Assessment of students' knowledge and understanding of mathematical concepts should provide evidence that they can compare, contrast, label, verbalize, and define concepts; identify and generate examples and counter-examples as well as properties of a given concept; recognize the various meanings and interpretations of concepts; and defend procedures and personal strategies. Students who have developed a conceptual understanding of mathematics can also use models, symbols, and diagrams to represent concepts. Appropriate assessment provides evidence of the extent to which students have integrated their knowledge of various concepts.

Problem Solving

Appropriate assessment of problem-solving skills is achieved by allowing students to adapt and extend the mathematics they know and by encouraging the use of strategies to solve unique and unfamiliar problems. Assessment of problem solving involves measuring the extent to which students use these strategies and knowledge and their ability to verify and interpret results. Students' ability to solve problems develops over time as a result of their experiences with relevant situations that present opportunities to solve various types of problems. Evidence of problem-solving skills is often linked to clarity of communication. Students demonstrating strong problem-solving skills should be able to clearly explain the process they have chosen, using appropriate language and correct mathematical notation and conventions.

Mathematics 30–2 Formula Sheet

Relations and Functions

Graphing Calculator Window Format

$$x: [x_{\min}, x_{\max}, x_{\text{scl}}]$$

$$y: [y_{\min}, y_{\max}, y_{\text{scl}}]$$

Exponents and Logarithms

$$y = a^x \leftrightarrow x = \log_a y$$

$$\log_b c = \frac{\log_a c}{\log_a b}$$

Laws of Logarithms

$$\log_b(M \cdot N) = \log_b M + \log_b N$$

$$\log_b\left(\frac{M}{N}\right) = \log_b M - \log_b N$$

$$\log_b(M^n) = n \log_b M$$

Exponential functions

$$y = a \cdot b^x$$

Logarithmic functions

$$y = a + b \cdot \ln x$$

Sinusoidal functions

$$y = a \cdot \sin(bx + c) + d$$

$$\text{Period} = \frac{2\pi}{b}$$

Quadratic equations

$$\text{For } ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Probability

$$n! = n(n-1)(n-2)\dots 3 \cdot 2 \cdot 1, \\ \text{where } n \in N \text{ and } 0! = 1$$

$${}_n P_r = \frac{n!}{(n-r)!}$$

$${}_n C_r = \frac{n!}{(n-r)!r!}$$

$${}_n C_r = \binom{n}{r}$$

$$P(A \cup B) = P(A) + P(B)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cap B) = P(A) \cdot P(B)$$

$$P(A \cap B) = P(A) \cdot P(B | A)$$

Logical Reasoning

A' Complement

\emptyset Empty set

\cap Intersection

\subset Subset

\cup Union