Assessment Standards & Exemplars

Mathematics 30–2

Alberta Provincial Diploma Examinations

Alberta

This document was written primarily for:

Students	✓
Teachers	✓ of Mathematics 30–2
Administrators	✓
Parents	
General Audience	
Others	

Distribution: This document is posted on the Alberta Education website.

Contents

Please note that if you cannot access one of the direct website links referred to in this document, you can find diploma examination-related materials on the Alberta Education website.

Introduction

This resource is designed to support the implementation of the *Alberta Mathematics Grades 10-12 Program of Studies*, which can be found on the Alberta Education website. Teachers are strongly encouraged to consult the program of studies for details about the philosophy of the program.

The examples shown in this document were chosen to illustrate the intent of the outcomes of Mathematics 30–2 but will not necessarily be assessed on a diploma examination in the manner shown. The examples provided are by no means exhaustive; they were intended to provide a profile of both acceptable achievement and excellent achievement. Some examples were developed and validated by classroom teachers of mathematics but have not been validated with students. Other examples were taken from previous diploma examinations. For more examples, please go to the *Quest A+* website.

To meet the outcomes of Mathematics 30–2, students will need access to an approved graphing calculator. In most classrooms, students will use a graphing calculator daily. Refer to the calculator policy in the *General Information Bulletin* or go to the Alberta Education website for a list of approved graphing calculators. Information about the diploma examinations each year can also be found in the *Mathematics 30–2 Information Bulletin*.

This document presents the current version of the curriculum and assessment standards. If you have comments or questions regarding this document, please contact Jenny Kim by email at Jenny.Kim@gov.ab.ca, by phone at 780-415-6127 (dial 310-0000 to be connected toll free).

The Provincial Assessment Sector would like to recognize and thank the many teachers throughout the province who helped to prepare this document. We would also like to thank the Programs of Study and Resources Sector and the French Language Education Services Branch for their input and assistance in reviewing these standards.

Standards for Mathematics 30–2

The word *and* used in the standards implies that both ideas should be addressed at the same time or in the same question.

The assessment standards for Mathematics 30–2 include an acceptable and an excellent level of performance.

Acceptable Standard

Students who attain the acceptable standard, but not the standard of excellence, will receive a final course mark between 50% and 79%, inclusive. Typically, these students have gained new skills and a basic knowledge of the concepts and procedures relative to the general and specific outcomes defined for Mathematics 30–2 in the program of studies. They demonstrate mathematical skills as well as conceptual understanding, and can apply their knowledge to familiar problem contexts.

Standard of Excellence

Students who attain the standard of excellence will receive a final course mark of 80% or higher. Typically, these students have gained a breadth and depth of understanding regarding the concepts and procedures, as well as the ability to apply this knowledge and conceptual understanding to a broad range of familiar and unfamiliar problem contexts.

General Notes

- The seven mathematical processes – C, CN, ME, PS, R, T, V – should be used and integrated throughout the outcomes.
- If technology [T] has not been specifically listed as a mathematical process for an outcome, teachers may, at their discretion, use it to assist students in exploring patterns and relationships when learning a concept. However, it is expected that technology will not be considered when assessing students' understandings of such outcomes.
- Most high school mathematics resources in North America use the letter *I* to represent the set of integers; however, students may encounter resources, especially at the post-secondary level, that use the letter *Z* to represent the set of integers. Both are correct.

Standards for Logical Reasoning

General Outcome

Develop logical reasoning.

General Notes:

- This topic is intended to develop students' numerical and logical reasoning skills, which are applicable to many everyday situations.

Specific Outcome 1

Analyze puzzles and games that involve numerical and logical reasoning, using problem-solving strategies. [**CN**, **ME**, **PS**, **R**]

Notes:

- This is an extension of Mathematics 20–2 Number and Logic Specific Outcome 2.
- The intent of this outcome is for students to explore games and puzzles to develop personal strategies, with an emphasis on the development of logical reasoning skills.
- A sample list of puzzles and games that could be used in working through this outcome is shown below. These games and puzzles could be integrated throughout the course as an alternative to devoting a fixed number of lessons to them.

Logic Puzzles/Games	**Strategic Puzzles/Games**
Tic Tac Chec	Chess
Mastermind	Cribbage
Sudoku	Nim
Kakuro	Battleship
Logic puzzles	Backgammon
Magic squares	Othello
Rush Hour	Sequence
Strimko	Blokus

- Questions will address the logic and reasoning processes within games or puzzles. Detailed knowledge of specific games will not be assessed on the diploma examination.
- The calendar in each issue of *Mathematics Teacher* may be a source of puzzle questions for class discussion or small group work.

(Refer to examples 1, 2, 3, 4, 5, 6, 7, and 8.)

Specific Outcome 2
Solve problems that involve the application of set theory. [**CN**, **PS**, **R**, **V**] [**ICT: C6–2.3**]

Notes:

- Students should be familiar with different classifications of numbers (e.g., number systems, prime and composite numbers, multiples of numbers, factors, etc.).
- Students should be familiar with verbal descriptions, symbols, and graphic organizers when describing sets and applying set theory.
- Teachers should be aware that Venn diagrams and logical reasoning symbols are also used in probability.
- Teachers should be aware that there are other set theory symbols that may be used; however, students should be familiar with the symbols on the formula sheet.
- In set theory, the word *or* is inclusive (i.e., it means “and/or”).

(Refer to examples 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, and 28.)

Acceptable Standard	Standard of Excellence
The student can	The student can also
• solve pattern problems involving logic or numerical reasoning	
• completely solve a puzzle involving logic	
• develop a successful strategy within the rules of a game	• describe a successful strategy within the rules of a game and explain why the strategy works
• integrate additional information to adapt a strategy	
• identify errors in a solution to a puzzle or in a strategy for winning a game	• identify and correct errors in a solution to a puzzle or in a strategy for winning a game
• create a variation on a puzzle or a game and develop an appropriate strategy	• justify a change in strategy for solving a puzzle or for winning a game when variations have been made to the puzzle or game
• use set notation correctly, including the logical reasoning symbols A', $\varnothing$, $\cap$, $\subset$, and $\cup$	
• correctly describe or identify complements of sets, the empty set, disjoint sets, subsets, and universal sets in context	• correctly describe or identify the result of an operation on two sets that involves complements (e.g., $A' \cup B$, $(A \cup B)'$, etc.)
• solve problems that involve the analysis of two sets within a universal set	• solve problems that involve the analysis of three sets within a universal set
• explain the reasoning used in solving set theory problems that involve two sets	• explain the reasoning used in solving set theory problems that involve three sets
• identify errors in a solution to a problem that involves two sets	• identify errors in a solution to a problem that involves three sets
	• identify and correct errors in a solution to a problem that involves sets
• participate in and contribute toward the problem-solving process for problems that require the application of logical reasoning studied in Mathematics 30–2	• complete the solution to problems that require the application of logical reasoning studied in Mathematics 30–2

Sample Questions

Students who achieve the acceptable standard should be able to answer all of the following questions, except for any part of a question labelled **SE**. Parts labelled **SE** are appropriate examples for students who achieve the standard of excellence.

Note: In the multiple-choice questions that follow, an asterisk (*) indicates the correct answer. Please be aware that the worked solutions shown are possible strategies; there may be other strategies that could be used.

Use the following information to answer questions 1 and 2.

Three rows of a pattern are shown below.

Row 1	$1 \times 8 + 1 = 9$
Row 2	$12 \times 8 + 2 = 98$
Row 3	$123 \times 8 + 3 = 987$

1. Row 5 of the pattern will be

A. $1\,234 \times 8 + 4 = 9\,876$
B. $1\,234 \times 8 + 5 = 9\,876$
C. $12\,345 \times 8 + 4 = 9\,876$
***D.** $12\,345 \times 8 + 5 = 98\,765$

2. If the number 8 in the pattern above is replaced by the number 9 as shown below, describe a pattern that could be used to calculate the value of row 7.

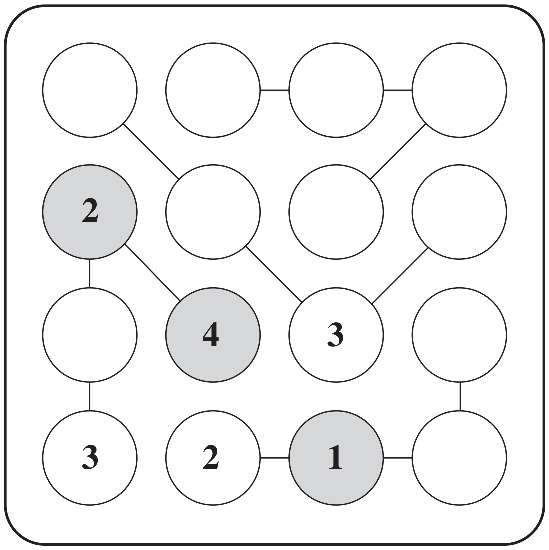
Row 1	$1 \times 9 + 1 = 10$
Row 2	$12 \times 9 + 2 = 110$
Row 3	$123 \times 9 + 3 = 1\,110$

Possible Solution:
The row number indicates the number of ones that occur in the answer, followed by a single zero. Therefore, the value of row 7 would be seven ones, followed by a single zero (i.e., 11 111 110). Alternatively, the value of row 7 could be calculated by recognizing the pattern in the operations.

Use the following information to answer question 3.

The goal of a particular puzzle is to fill the circles in a grid with the numbers 1, 2, 3, and 4 so that no numbers are repeated in any row, column, or set of connected circles.

The three entries in the grey circles were given to start the puzzle. Jerome has already completed three entries shown in the white circles, but he has made an error.



3. a. Identify the error that Jerome made in his solution to the puzzle.

Possible Solution:
The error that Jerome made was his entry of the number 3 in row 3, column 3.

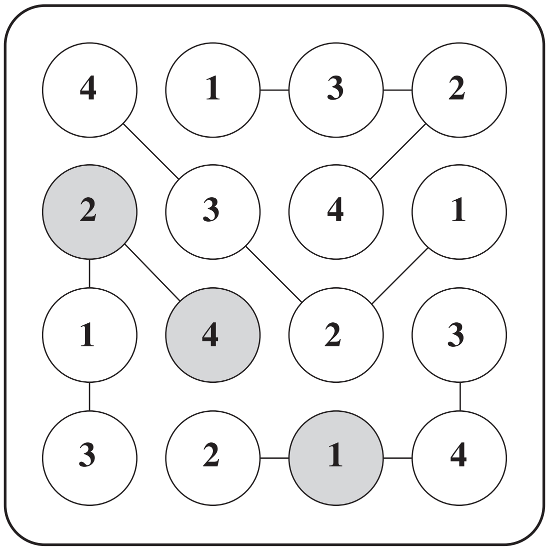
b. Explain why this entry is incorrect.

Possible Solution:
If Jerome's entries were correct, then the entry in row 3, column 1 must be a 1, and the entry in row 3, column 4 must be a 2. However, since the last three circles in row 4 are connected to the circle in row 3, column 4, this will mean that one of the rules of the game is violated because a set of connected circles has repeated numbers (i.e., two 2's).

SE

c. Correct the error that Jerome made and complete the puzzle.

Solution:



Note: This question could be adapted to be an interactive item on a digital test.

Use the following information to answer numerical-response question 4.

The goal of a particular two-player game is to be the first player to create a line of four adjacent rectangles containing the same letter. To play, each player takes turns placing their first initial somewhere on a six-by-six grid. Margaret and Gerda have started playing this game, as shown on the grid below.

Column (across), Row (down)

Row	1	2	3	4	5	6
1	G					
2		G				
3			G	G		
4			M	M	G	
5		M				
6	M	M	G			

Numerical Response

4. It is Margaret's turn, and she determines that she can guarantee a win by placing the letter M in the rectangle at

Row __________

Column __________

Solution:
42

If Margaret places her next M in row 4, column 2, she will have two sets of three M's available, giving her two options for winning when placing her next M. Gerda can block only one of them and cannot create a line of four adjacent G's on her next turn, so Margaret is guaranteed to win.

Note: This question could be adapted to be an interactive item on a digital test.

Use the following information to answer question 5.

Four girls – Alice, Brenda, Cathy, Dianna – go to a recreation centre in a large city. Each girl participated in a different activity – Weight Training, Skating, Jogging, Swimming – and each girl owns a backpack of a different colour – Blue, Pink, Green, Red. The four clues below provide information about the activity in which each girl participated and the colour of each girl's backpack.

- Cathy does not have a red backpack and needed to dress warmly for her activity.
- The swimmer who owned the red backpack was not Brenda.
- The girl who carried the green backpack was jogging.
- While preparing for her jog, Dianna watched her friend take her skates out of her blue backpack.

	Weight Training	**Skating**	**Jogging**	**Swimming**	**Blue**	**Pink**	**Green**	**Red**
Alice								
Brenda								
Cathy								
Dianna								

Blue				
Pink				
Green				
Red				

5. Determine the activity and backpack colour for each girl.

Possible Solution:

	Weight Training	Skating	Jogging	Swimming
Alice	✗	✗	✗	✓
Brenda	✓	✗	✗	✗
Cathy	✗	✓	✗	✗
Dianna	✗	✗	✓	✗
Blue	✗	✓	✗	✗
Pink	✓	✗	✗	✗
Green	✗	✗	✓	✗
Red	✗	✗	✗	✓

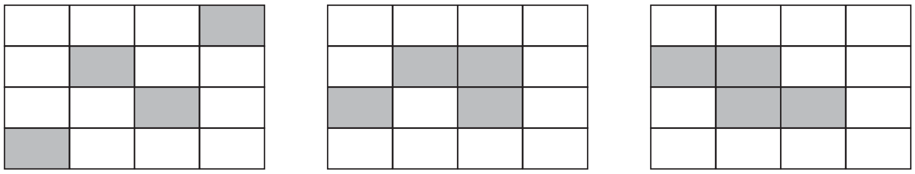
	Blue	Pink	Green	Red
Alice	✗	✗	✗	✓
Brenda	✗	✓	✗	✗
Cathy	✓	✗	✗	✗
Dianna	✗	✗	✓	✗

Alice participated in swimming and owns a red backpack. Brenda participated in weight training and owns a pink backpack. Cathy participated in skating and owns a blue backpack. Dianna participated in jogging and owns a green backpack.

Note: This question could be adapted to be an interactive item on a digital test.

Use the following information to answer question 6.

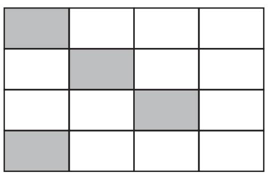
A pattern of pictures is shown below. In Step 2, each shaded square has stayed in the same place as in Step 1 **or** moved to a square horizontally, vertically, or diagonally adjacent to its location in Step 1. The shaded square undergoes the same movement in each subsequent step.



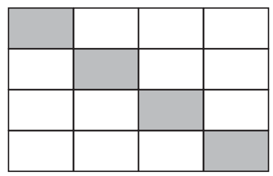
Step 1 **Step 2** **Step 3**

6. Which of the following pictures is next in the pattern?

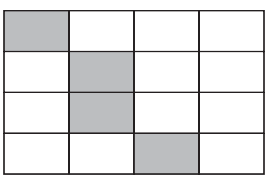
***A.**



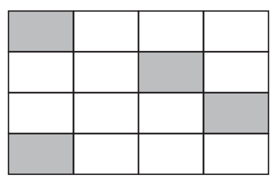
B.



C.



D.



Possible Solution:
The shaded box in the first column is moving upward in each step. The shaded boxes in the second and third columns remain stationary. The remaining shaded box, which started in column 4, is moving diagonally down and to the left in each step.

Use the following information to answer numerical-response question 7.

A student makes the following statement.

"VOTE compares to VETO as the number 8570 compares to the number __________."

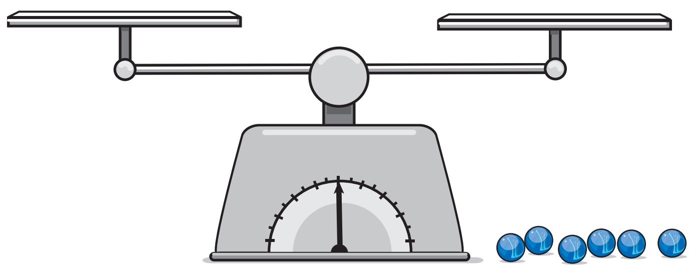
Numerical Response

7. The 4-digit number that completes the statement above is __________.

Solution:
8075

Use the following information to answer question 8.

Six marbles are identical in size, shape, and colour, but one of them is heavier than the others.



8. If you are able to use a balance scale, shown above, only two times, then how could you determine which marble is heavier than the others?

Possible Solution:
Place three marbles on each side of the scale to determine which set of marbles is heavier. Discard the lighter set of marbles. From the three marbles left, place one on each side of the scale and set the last marble aside. If the scale is unbalanced, you know which marble is heavier. If the scale is balanced, the marble you set aside is the heavier marble.

Use the following information to answer questions 9 and 12 and numerical-response questions 10 and 11.

Students in a particular high school were surveyed to determine the courses in which they were currently enrolled. The table below represents the data that were collected.

Courses	Number of Students
Math only	28
Art only	33
Math and Art	17
Neither course	20

9. The number of students in the universal set is

A. 61
B. 64
C. 78
***D.** 98

Numerical Response

10. The number of students taking Art is __________.

Solution:
50

There are 33 students who take Art only and 17 students who take both Math and Art.

Numerical Response

11. The number of students **not** taking Math is __________.

Solution:
53

There are 33 students who take Art only and 20 students who take neither Math nor Art.

12. The number of students taking Math or Art is

A. 17
B. 61
*__C.__ 78
D. 98

Use the following information to answer question 13.

There are 35 students in John's homeroom class. There are 5 students who take English and Biology, and 7 students who take neither of these subjects. There are 3 more students taking English only than there are students taking Biology only.

13. **a.** The number of students in John's homeroom who take Biology only is

*__A.__ 10
B. 13
C. 15
D. 20

b. The number of students in John's homeroom who do **not** take both English and Biology is

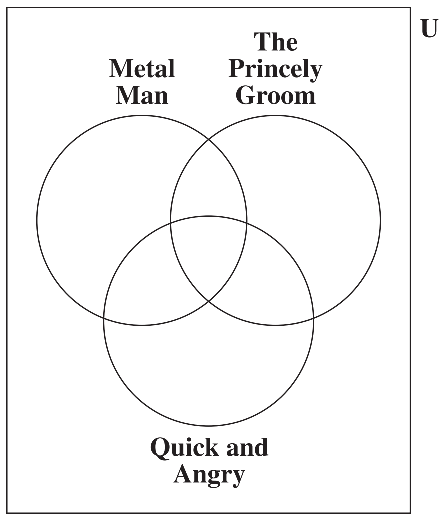
A. 5
B. 7
C. 23
*__D.__ 30

Use the following information to answer questions 14, 16, and 17 and numerical-response question 15.

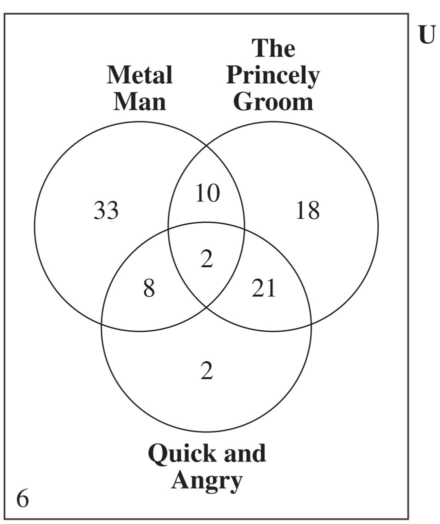
A group of 100 students was surveyed about movies that they have seen, as shown below.

2 people saw all three movies
12 people saw "Metal Man" and "The Princely Groom"
53 people saw "Metal Man"
10 people saw "Metal Man" and "Quick and Angry"
18 people saw "The Princely Groom" only
23 people saw "The Princely Groom" and "Quick and Angry"
6 people did not see any of the three movies

Jason started to organize the results in the Venn diagram shown below.



Completed Venn diagram to answer questions 14 to 17



SE **14.** The number of students who saw “The Princely Groom” is

A. 18
B. 20
***C.** 51
D. 53

Numerical Response

SE **15.** The number of students who saw “Metal Man” and “The Princely Groom” but **not** “Quick and Angry” is __________.

Solution:
10

SE **16.** The number of students who saw “Metal Man” only is

A. 20
***B.** 33
C. 51
D. 53

SE **17.** The number of students who saw “Metal Man” or “Quick and Angry” is

A. 10
B. 43
***C.** 76
D. 98

Use the following information to answer questions 18 and 19.

Two Sets

A = {prime numbers less than 20}
B = {factors of 20}

18. The set representing the union of sets A and B is

A. {2, 5}
B. {2, 4, 5, 10}
***C.** {1, 2, 3, 4, 5, 7, 10, 11, 13, 17, 19, 20}
D. {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

19. The set representing $A \cap B$ is

***A.** {2, 5}
B. {2, 4, 5, 10}
C. {1, 2, 3, 4, 5, 7, 10, 11, 13, 17, 19, 20}
D. {1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20}

Use the following information to answer numerical-response question 20.

Four Sets

Set	Set Description
1	{2, 3, 8, 9, 15, 16}
2	{2, 3, 5, 7, 11, 13, 17, 19}
3	{multiples of 3 between 0 and 20}
4	{odd numbers between 0 and 20}

Numerical Response

20. A pair of sets that will produce the intersection {3, 9, 15} are numbered, in any order, _____ and _____.

(There is more than one correct answer.)

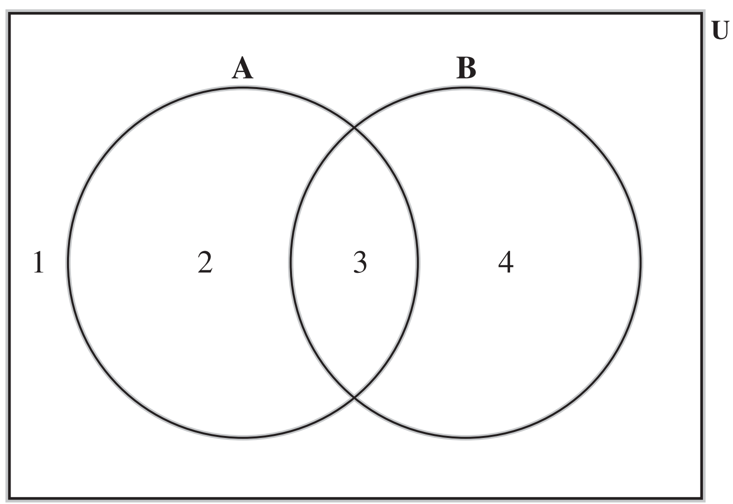
Possible Solutions:
13, 31; 14, 41; 34, 43

21. Which of the following rows describes two sets that are disjoint?

Row	Set 1	Set 2
A.	People who drink coffee	People who do not drink tea
B.	People who have a home phone line	People who have a cellular phone line
C.	People who are left-handed	People who own a computer
***D.**	People who live in Calgary	People who do not live in Alberta

Use the following information to answer numerical-response question 22.

A Venn diagram is used to illustrate the relationship between sets A and B. The regions of the Venn diagram are numbered 1 through 4, as shown below.



Numerical Response

SE **22.** The regions of the Venn diagram that must be shaded to represent $(A \cap B)'$ are numbered, in any order, __________.

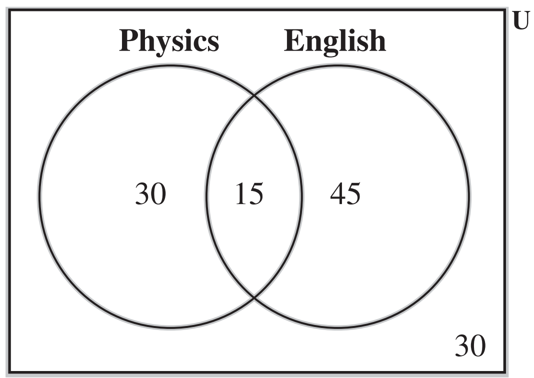
Possible Solutions:
124, 142, 214, 241, 412, 421

Use the following information to answer question 23.

In a school of 120 students:

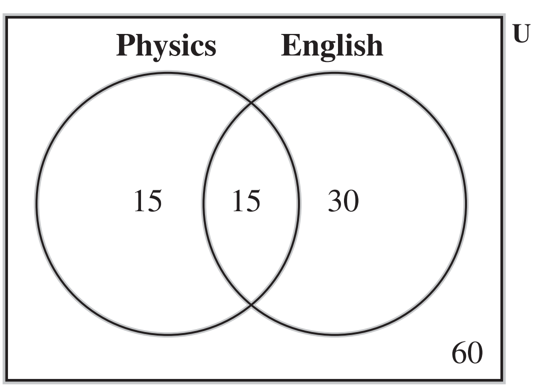
15 students took Physics and English
45 students took English
30 students took Physics

Bobby incorrectly summarized the data in a Venn diagram, which is shown below.



SE **23.** Identify the regions of Bobby's Venn diagram that have incorrect entries, and describe the errors that Bobby made. Make changes to the Venn diagram to show the correct entries.

Possible Solution:
When completing the Venn diagram, Bobby made errors in the regions that represent the number of students taking Physics only and the number of students taking English only. The number of students taking both courses is 15, and these students are already counted in the students taking Physics and the students taking English. Consequently, the number of students taking Physics only should be 30 – 15, or 15 students. The number of students taking English only should be 45 – 15, or 30 students. This will make the region within the universal set, but outside the circles, become 120 – (15 + 15 + 30), or 60 students. The correct Venn diagram is shown below.



Use the following information to answer questions 24 and 25.

Three Sets

R = {natural numbers less than 50}

S = {even numbers}

T = {10, 20, 30, 40}

24. Which of the following statements is true for sets R, S, and T?

A. $R \subset S$

B. $R \subset T$

C. $S \subset R$

***D.** $T \subset R$

SE

25. Which of the following statements is **not** true for sets R, S, and T?

A. $T \subset (R \cap S)$

B. $T \subset (R \cap T)$

***C.** $(R \cap S) \subset T$

D. $(R \cap T) \subset T$

Use the following information to answer question 26.

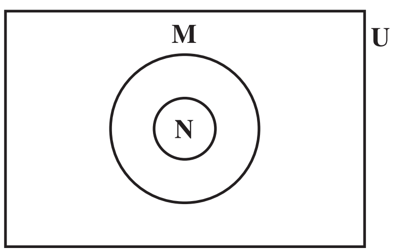
A student suggests that for any set A, the following statements are true.

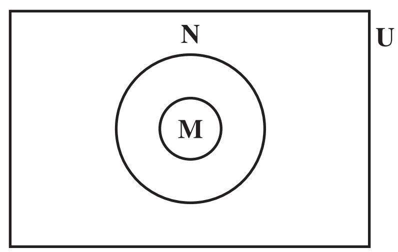
$$A \cup \varnothing = A$$
$$A \cap \varnothing = A$$

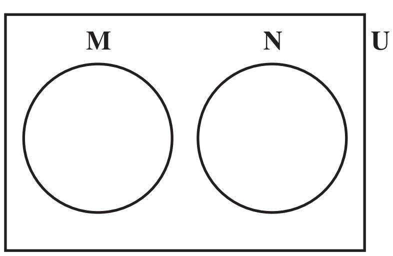
26. Is this student correct or incorrect? Use an example or a visual representation in your explanation.

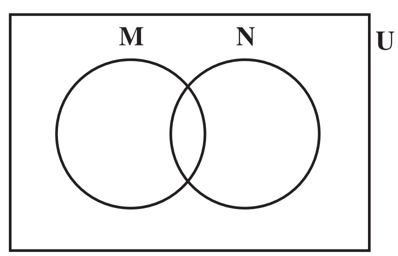
Possible Solution:
The student is correct that $A \cup \varnothing = A$. No elements are added to A by combining it with the empty set, so the initial set A remains after the union. For example, if $A = \{2, 3, 4\}$, and it is combined with a set that has no elements, the result will be $\{2, 3, 4\}$. The student is incorrect that $A \cap \varnothing = A$. Since the empty set has no elements, it cannot have any elements in common with A, so $A \cap \varnothing = \varnothing$. For example, if $A = \{2, 3, 4\}$, none of these elements occurs in $\varnothing$; there are no common elements, meaning the set of common elements is $\varnothing$.

27. Which of the following Venn diagrams illustrates $M \cap N = \varnothing$ for all sets M and N?

A.


B.


***C.**


D.


28. Which of the following phrases describes an empty set?

A. Common factors of 3 and 7

B. Prime numbers that are even

C. Multiples of 5 that are less than 10

***D.** Perfect squares less than 20 that are divisible by 5

Standards for Probability

General Outcome

Develop critical thinking skills related to uncertainty.

General Notes:

- The concept of sample space could be introduced at the start of this topic to form the basis for all probability work to follow. Sample spaces may exceed two events.
- In probability, the word *or* is inclusive (i.e., it means "and/or").
- Teachers should be aware that probability problems could include Venn diagrams and logical reasoning symbols.
- Teachers should be aware that specific outcomes 4, 5, and 6 do not require the simplification of factorial expressions (e.g., Simplify $\frac{n!}{(n-2)!}$).

Specific Outcome 1

Interpret and assess the validity of odds and probability statements. **[C, CN, ME]**

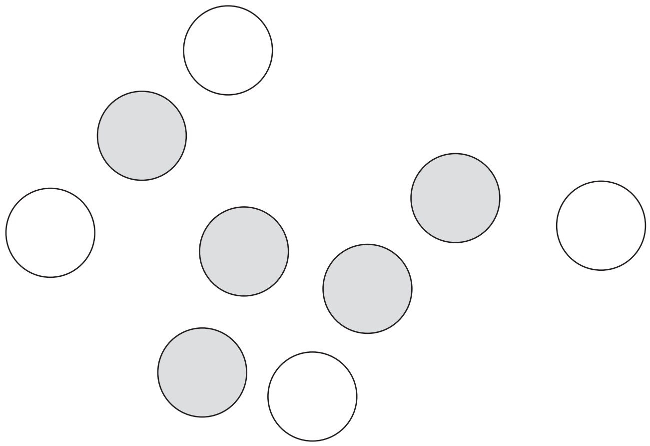
Notes:

- Teachers may wish to use the analogy that odds can be considered "part-part" and probability can be considered "part-whole."
- Students should be familiar with the terms *odds for*, *odds in favour of*, and *odds against*.
- Odds and probabilities can be represented in many ways, including but not limited to the following:

$$\text{Odds in favour of A} = \text{\# of outcomes for A} : \text{\# of outcomes against A}$$

$$P(\text{event A}) = \frac{\text{\# of favourable outcomes}}{\text{total \# of outcomes}}$$

- Concrete models, like the one shown below, can be used to model probability and odds.



5 grey parts
4 white parts
The whole consists of 9 parts.

(Refer to examples 1, 2, 3, 4, and 5.)

Specific Outcome 2
Solve problems that involve the probability of mutually exclusive and non–mutually exclusive events. **[CN, PS, R, V] [ICT: C6–2.3]**

Notes:

- Students can use a variety of strategies to solve mutually exclusive and non–mutually exclusive problems. Some possible strategies include using graphic organizers (e.g., tree diagrams, Venn diagrams, tables, organized lists) and probability formulas.
- Since complementary events are also mutually exclusive, this specific outcome does include the concept of complementary events.
- Problems for this outcome should be limited to a maximum of two events.

(Refer to examples 6, 7, 8, 9, 10, 11, and 30d.)

Specific Outcome 3
Solve problems that involve the probability of two events. **[CN, PS, R]**

Notes:

- This outcome includes a study of independent events and dependent events.
- Students can use a variety of strategies to determine probabilities of independent events and dependent events. Some possible strategies include using graphic organizers (e.g., tree diagrams, tables, organized lists) and probability formulas.
- Teachers may wish to make connections between this outcome and specific outcomes 4, 5, and 6.

(Refer to examples 12, 13, 14, 15, 24, and 30c.)

Specific Outcome 4
Solve problems that involve the fundamental counting principle. **[PS, R, V] [ICT: C6–2.3]**

Notes:

- Students can use a variety of organizational strategies (e.g., tree diagrams, charts, and other visual cues) to solve problems involving the fundamental counting principle.
- Factorial notation could be introduced in the development of this outcome.
- Teachers may wish to make connections between this outcome and specific outcome 3.
- Teachers should be aware that repetition of elements is not considered a restriction when distinguishing between the acceptable standard and the standard of excellence.

(Refer to examples 16, 17, 18, 19, 22, 23, and 24.)

Specific Outcome 5
Solve problems that involve permutations. **[ME, PS, R, T, V]**

Notes:

- Factorial notation could be discussed in this outcome in conjunction with the ${}_nP_r$ formula. However, other strategies can also be used to solve problems involving permutations.
- When questions involving permutations with identical elements are solved, all of the elements within the context should be arranged. These questions will involve at most one restriction.
- Simple 2-D pathways (e.g., no gaps in the grid, no overlapping regions) are applications of permutations with identical elements.
- Circular and ring permutations are beyond the scope of Mathematics 30–2.
- Teachers should consider probability problems that involve permutations. This is a connection to specific outcome 3.

(Refer to examples 20, 21, 22, 23, 24, and 25.)

Specific Outcome 6
Solve problems that involve combinations. **[ME, PS, R, T, V]**

Notes:

- Students should be familiar with both notations for combinations, as shown on the formula sheet.
- A problem that requires both permutations and combinations is beyond the scope of Mathematics 30–2.
- Teachers should consider probability problems that involve combinations. This is a connection to specific outcomes 2 and 3.

(Refer to examples 25, 26, 27, 28, 29, and 30.)

Acceptable Standard	Standard of Excellence
The student can	The student can also
• determine the probability of an event	• provide an explanation for the validity of a probability statement
• determine odds in favour of or odds against an event	• provide an explanation for the validity of an odds statement
• determine the odds in favour of an event given the odds against the event or vice versa	
• express odds in favour of or odds against an event as a probability	• express probability of an event as odds in favour of or odds against
• distinguish between mutually exclusive events and non-mutually exclusive events	
• determine $P(A \cup B)$ for events that are mutually exclusive	• determine $P(A \cup B)$ for events that are non-mutually exclusive
• determine $P(A)$ given $P(A \cup B)$ and $P(B)$ for mutually exclusive events	• determine $P(A)$ given $P(A \cup B)$, $P(A \cap B)$, and $P(B)$ for non-mutually exclusive events
• interpret a model that represents any combination of mutually exclusive and non-mutually exclusive events	• represent events that are non-mutually exclusive using a graphic organizer
• describe or identify complementary events	
• determine the probability of the complement of an event, given the probability of the event	
• distinguish between independent and dependent events	
• determine $P(A \cap B)$ for independent events	• determine $P(A)$ when given $P(A \cap B)$ and $P(B)$ for independent events
• determine $P(A \cap B)$ for dependent events, given the order of the events	• determine $P(A \cap B)$ for dependent events when the order of the events is not given
	• determine $P(A)$ when given $P(A \cap B)$ and $P(B \mid A)$ for dependent events
	• determine $P(A \cap B)$ for problems that involve two or three cases
• apply the fundamental counting principle to problems with at most one restriction	• apply the fundamental counting principle to problems with more than one restriction
	• apply the fundamental counting principle to problems that involve two or three cases (e.g., at least, at most, or)

• determine the number of permutations of n objects with some identical elements taken n at a time	• determine the number of permutations of n objects with some identical elements taken n at a time, with one restriction
• determine the number of permutations of n distinct objects taken r at a time	• solve permutation problems that involve two or three cases (e.g., at least, at most, or)
• determine the number of permutations of n objects taken r at a time, with at most one restriction	• determine the number of permutations of n objects taken r at a time, with more than one restriction
• solve combination problems that involve a single case	• solve combination problems that involve two or three cases (e.g., at least, at most, or)
• distinguish between problems that describe permutations and problems that describe combinations	
• solve probability problems that describe permutations	
• solve probability problems that describe single combinations in the numerator $\left(\text{e.g., } \frac{_5C_3}{_{11}C_3}\right)$	• solve probability problems that involve two or three combinations in the numerator $\left(\text{e.g., } \frac{_5C_3 \cdot {_4C_2}}{_9C_5}\right)$
• participate in and contribute toward the problem-solving process for problems that require the analysis of probability studied in Mathematics 30–2	• complete the solution to problems that require the analysis of probability studied in Mathematics 30–2

Sample Questions

Students who achieve the acceptable standard should be able to answer all of the following questions, except for any part of a question labelled **SE**. Parts labelled **SE** are appropriate examples for students who achieve the standard of excellence.

Note: In the multiple-choice questions that follow, an asterisk (*) indicates the correct answer. Please be aware that the worked solutions shown are possible strategies; there may be other strategies that could be used.

1. The odds in favour of the Renegades winning the season final in a football league are listed as 10 : 7. The odds against the Renegades winning the season final are

A. 3 : 7
B. 3 : 10
***C.** 7 : 10
D. 10 : 3

Numerical Response

2. Statistics show that 6 out of 25 car accidents are weather-related. The odds in favour of a car accident being weather-related can be expressed in the form $a : b$. The values of a and b are, respectively, _____ and _____.

Solution:
6 and 19

SE

3. A class of 35 students has 17 males. One student will be selected at random from the class. Jeanette suggested that the odds for selecting a male student are 17 : 35. Is Jeannette correct? Justify your answer.

Possible Solution:
Jeanette is incorrect in stating that the odds are 17 : 35. Odds are measured as one "part" versus the "other part." In this situation, there are two complementary parts: male and female. As such, the odds in favour of selecting a male should be stated as the ratio of the number of males to the number of females. Therefore, the odds for selecting a male student would be 17 : 18.

Use the following information to answer question 4.

A television game show has listed the following odds in favour of winning for three of their games.

Game	Odds of Winning
Flip'em	1 : 3
Central Eye	2 : 5
Minefield	1 : 4

4. a. What is the probability of winning the Flip'em game?

Solution:

$\frac{1}{4}$ **or** 0.25

b. Which of the three games is a contestant **most likely** to win? Justify your answer.

Possible Solution:
The probabilities of winning for each game are

Flip'em	0.25
Central Eye	0.29
Minefield	0.20

The game with the highest probability of winning is Central Eye; therefore, this is the game that a contestant is **most likely** to win.

Use the following information to answer question 5.

> Climate change is a major concern for the insurance industry since weather-related disasters are becoming less predictable and more probable. To avoid bankruptcy, insurance managers need to anticipate the odds in favour of weather-related disasters. As an island nation, the United Kingdom (UK) must be aware of the effect of changing climates on oceans and other bodies of water. Historically, the odds in favour of a 100-year flood plain experiencing flooding in any given year are 1 : 99. When considering flood insurance on a 100-year flood plain, it has been estimated by scientific agencies that *"… the world with climate change has a 20 percent greater chance of experiencing those floods than the world without."* (*The New York Times*).

SE

5. Based on the information from *The New York Times*, Randall claimed that because of climate change, the new odds in favour of a flood occurring on a 100-year flood plain in the UK would be 21 : 99. Explain why you agree or disagree with Randall's claim. Include an explanation of odds and probability in your response.

Possible Solution:

I disagree with Randall's claim that the new odds in favour of a flood on a 100-year floodplain in the UK will be 21 : 99. The original odds were 1 : 99, which means that the probability of a flood occurring is $\frac{1}{100}$ or 0.01. The quote suggests that there is a "20% greater chance" of a flood occurring. This can be interpreted in two different ways. The quote can mean that the new probability will be the old probability increased by 20%, to 0.21 (0.01 + 0.20). In this case, the odds in favour of the flood are 21 : 79. The quote could also mean that the new probability will be 0.01 • 1.20 (a 20% increase) = 0.012, which means that the odds in favour of a flood are 12 : 988 = 3 : 247. Neither of these odds statements matches Randall's claim; therefore, Randall's claim is incorrect.

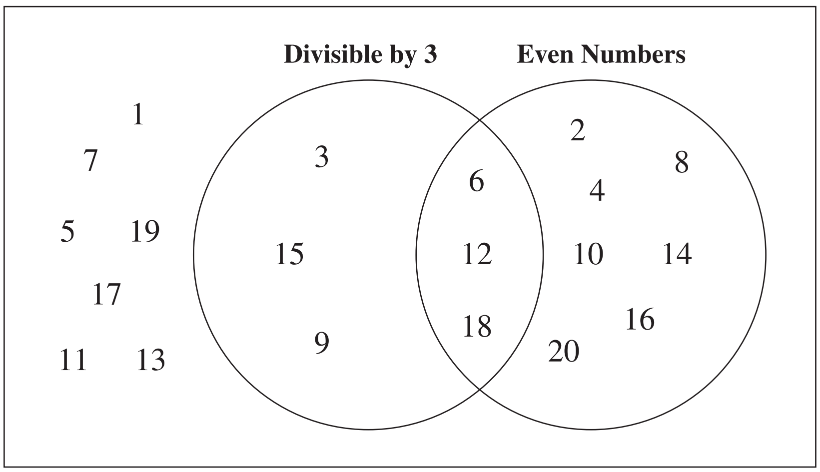
Use the following information to answer question 6.

For the set of whole numbers from 1 to 20 inclusive, Theresa knows that some numbers are divisible by 3 and some numbers are even. She is going to write each number on a different ball and place the balls in a box.

SE

6. If one ball is randomly selected from the box, what is the probability that the number written on it is divisible by 3 or is an even number?

Possible Solution:



The favourable outcomes are the outcomes within the circles, so the number of favourable outcomes is 13.

$P(\text{Divisible by 3 or Even Number}) = \frac{13}{20}$

or

$P(\text{Divisible by 3} \cup \text{Even number})$
$= P(\text{Divisible by 3}) + P(\text{Even Number}) - P(\text{Divisible by 3} \cap \text{Even Number})$

$= \frac{6}{20} + \frac{10}{20} - \frac{3}{20}$
$= \frac{13}{20}$

7. A particular traffic light at the outskirts of a town is red for 30 s, green for 25 s, and yellow for 5 s in every minute. When a vehicle approaches the traffic light, the probability that the light will be red or yellow is

*A. $\frac{7}{12}$

B. $\frac{1}{2}$

C. $\frac{1}{12}$

D. $\frac{1}{24}$

Use the following information to answer numerical-response question 8.

Malaga, Spain, lies in a region of Europe known as the Costa Del Sol (Coast of the Sun). The probability of sunshine on any given day in the region is approximately 0.89.

Numerical Response

8. In a non-leap year of 365 days, the average number of days of the year that a tourist could expect to experience weather other than sunshine, to the nearest whole number, is __________ days.

Possible Solution:

$$1 - 0.89 = 0.11$$
$$0.11 \times 365 = 40.15$$
$$\approx 40 \text{ days}$$

Use the following information to answer numerical-response question 9.

Some possible events for rolling a regular six-sided die are listed below.

1 An even number
2 A number less than 3
3 A number that is a multiple of 3
4 A number that is greater than or equal to 2

Numerical Response

9. From the list above, the two events that are mutually exclusive are numbered _____ and _____.

Solution:
23 **or** 32

Use the following information to answer question 10.

A recent survey determined that 85% of a population watches TV at least once a day, 35% of the population uses a computer at least once a day, and 25% of the population do both.

SE

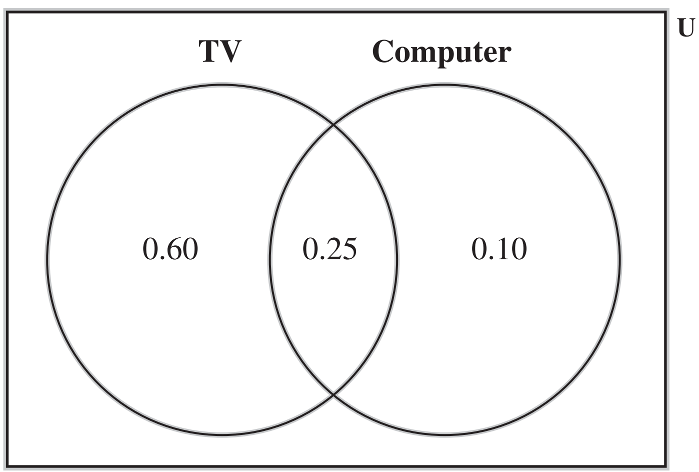
10. a. What is the probability that a person chosen at random from the population watches TV at least once a day or uses a computer at least once a day?

Possible Solution:

$P(\text{TV}) = 0.85$
$P(\text{Computer}) = 0.35$
$P(\text{TV} \cap \text{Computer}) = 0.25$

$$\begin{aligned} P(\text{TV} \cup \text{Computer}) &= P(\text{TV}) + P(\text{Computer}) - P(\text{TV} \cap \text{Computer}) \\ &= 0.85 + 0.35 - 0.25 \\ &= 0.95 \end{aligned}$$

or



$P(\text{TV} \cup \text{Computer}) = 0.60 + 0.25 + 0.10 = 0.95$

b. Are the events of watching TV at least once a day and using the computer at least once a day mutually exclusive events? Justify your answer.

Possible Solution:

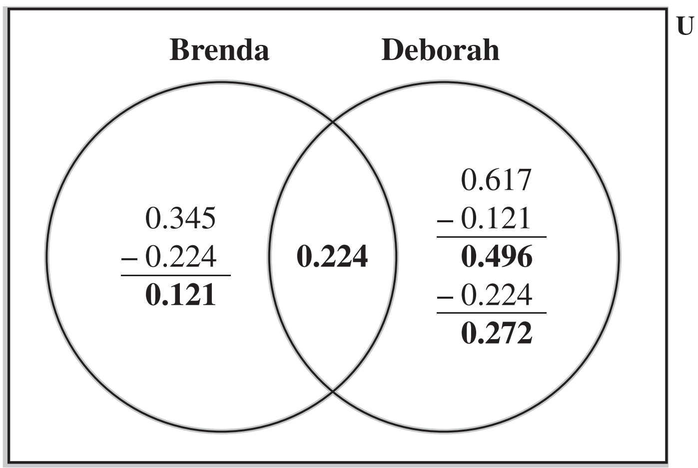
These events are not mutually exclusive because some of the survey participants do both activities.

Use the following information to answer question 11.

The probability of Brenda getting a hit in a baseball game is 0.345. The probability of Brenda or Deborah getting a hit during the game is 0.617. The probability of both Brenda and Deborah getting hits during the game is 0.224.

SE **11.** Determine the probability of Deborah getting a hit in the game.

Possible Solution:



$P(\text{D}) = 0.224 + 0.272$
$= 0.496$

or

$P(\text{B} \cup \text{D}) = P(\text{B}) + P(\text{D}) - P(\text{B} \cap \text{D})$
$0.617 = 0.345 + \text{P(D)} - 0.224$
$0.496 = P(\text{D})$

Use the following information to answer question 12.

Alan places five white marbles and five black marbles into a bag. He then performs the two experiments described below to select two marbles from the bag.

First Experiment
One marble is selected from the bag and replaced before a second marble is selected.

Second Experiment
One marble is selected from the bag and **not** replaced before a second marble is selected.

The following two events are the same for each experiment:

Event X: The first marble selected is black.
Event Y: The second marble selected is white.

12. *In the first experiment, Event X and Event Y are* ___***i***___ *, and in the second experiment, Event X and Event Y are* ___***ii***___ .

The statement above is completed by the information in row

Row	***i***	***ii***
A.	dependent	independent
B.	dependent	dependent
C.	independent	independent
***D.**	independent	dependent

13. A box contains 6 blue balls and 4 red balls. A student draws 2 balls from the box, one after the other, without replacement. The probability, to the nearest hundredth, that the first ball drawn is blue and the second ball drawn is red is __________.

Possible Solution:

$$P(\text{blue} \cap \text{red}|\text{blue}) = P(\text{blue}) \cdot P(\text{red}|\text{blue})$$
$$= \frac{6}{10} \cdot \frac{4}{9}$$
$$= 0.27$$

14. Based on previous performance, the probability of a particular baseball team winning any game is $\frac{4}{5}$.

a. The probability that this team will win their next 2 games is

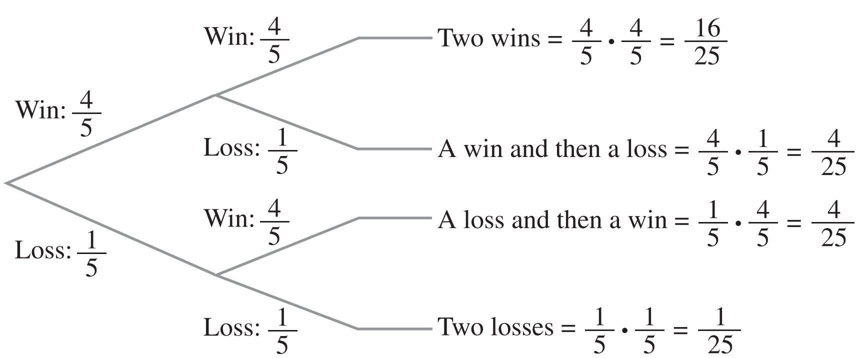
A. $\frac{1}{5}$

B. $\frac{4}{5}$

C. $\frac{1}{25}$

***D.** $\frac{16}{25}$

Possible Solution:



The probability that the baseball team wins their next two games is $\frac{16}{25}$.

SE

b. What is the probability that the team will win 1 game and lose 1 game out of the next 2 games?

Possible Solution:

$$P(\text{Win} \cap \text{Lose}) = P(\text{Win 1st} \cap \text{Lose 2nd}) + P(\text{Lose 1st} \cap \text{Win 2nd})$$

$$= \left(\frac{4}{5} \cdot \frac{1}{5}\right) + \left(\frac{1}{5} \cdot \frac{4}{5}\right)$$

$$= \frac{4}{25} + \frac{4}{25}$$

$$= \frac{8}{25}$$

Use the following information to answer question 15.

> From a particular bag that contains tiles, one tile is selected and the colour is recorded. From a second bag that contains marbles, one marble is selected and the colour is recorded. The probability of randomly selecting a blue tile from the first bag is 0.62. The probability of randomly selecting a blue tile from the first bag and a blue marble from the second bag is 0.46.

SE

15. The probability, to the nearest hundredth, of selecting a blue marble from the second bag is __________.

Possible Solution:

$$P(\text{Blue tile} \cap \text{Blue marble}) = P(\text{Blue tile}) \cdot P(\text{Blue marble})$$
$$0.46 = 0.62 \cdot P(\text{Blue marble})$$
$$0.74 = P(\text{Blue marble})$$

16. A hotel offers free breakfast to its guests. One morning the hotel has 3 different kinds of juice, 4 different kinds of cereal, and 2 different types of pastries available. If Tim must choose one kind of juice, one kind of cereal and one type of pastry, how many different possible breakfasts can be ordered?

Possible Solution:

$$\underset{\textbf{juice}}{\underline{\quad 3 \quad}} \cdot \underset{\textbf{cereal}}{\underline{\quad 4 \quad}} \cdot \underset{\textbf{pastry}}{\underline{\quad 2 \quad}} = 24$$

Use the following information to answer numerical-response question 17.

A new licence plate in Alberta consists of three letters followed by four digits. Letters are chosen from a list of 23 acceptable letters that may be repeated.

Maureen wants the first letter on her licence plate to be an M, which is an acceptable letter, and she also wants the four digits to match the last four digits of her cell phone number in the same order.

Numerical Response

SE

17. The number of licence plates that will meet Maureen's criteria is __________.

Possible Solution:
1 • 23 • 23 • 1 • 1 • 1 • 1 = 529

18. a. Determine the number of 4-digit numbers that can be created using the digits 0 to 9 without repetition.

Possible Solution:
9 • 9 • 8 • 7 = 4 536

SE

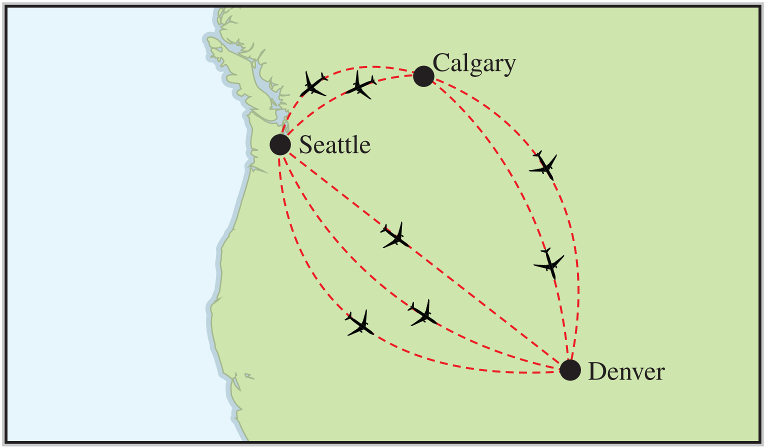
b. Determine the number of 6-digit odd numbers that can be created using the digits 0 to 9 without repetition. Describe any restrictions that exist.

Possible Solution:
8 • 8 • 7 • 6 • 5 • 5 = 67 200

The sixth digit is restricted if the number is going to be odd. The first digit is also restricted – it cannot be 0, nor can it be the same digit as the sixth digit.

Use the following information to answer numerical-response question 19.

Julian is planning a trip from Calgary to Denver. The map below shows the different flight options for a particular airline.



Numerical Response

SE

19. If Julian must fly with this airline, then how many different flight options are possible?

Possible Solution:
Calgary to Denver **or** Calgary to Seattle to Denver
$2 + 2 \cdot 3$
$= 8$

20. **a.** Determine the number of distinct arrangements of all the letters in the word TATTOO.

Possible Solutions:

$\frac{6!}{3!2!} = 60$ **or** $\frac{{}_6P_6}{3!2!} = 60$ **or** $\frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{3!2!} = 60$

SE

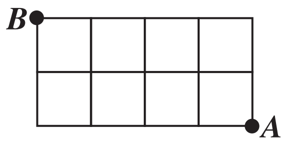
b. How many distinct arrangements of all the letters in the word TATTOO start with the letter T?

Possible Solution:

$\frac{3 \cdot 5!}{3!2!} = 30$

21. Determine the number of different possible routes that Tyler can travel from Point A to Point B if he travels only north or west.

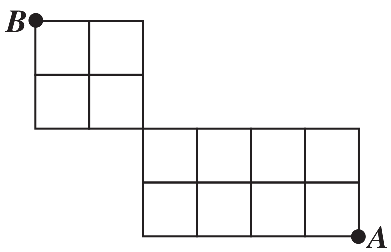
a.



Possible Solution:

$$\frac{6!}{4!2!} = 15$$

b.



Possible Solution:

$$\frac{6!}{4!2!} \cdot \frac{4!}{2!2!} = 90$$

22. Determine the number of distinguishable 3-letter arrangements using all the letters in the word **DIPLOMA**.

Possible Solutions:

$${}_nP_r = \frac{n!}{(n-r)!}$$

$${}_7P_3 = \frac{7!}{(7-3)!}$$

$${}_7P_3 = 210$$

or

$${}_7P_3 = 210$$

or

$7 \cdot 6 \cdot 5 = 210$

23. A 7-player volleyball team must stand in a line for a picture.

a. Determine the number of different player arrangements that can be made for the picture.

Possible Solutions:
$7! = 5\ 040$ **or** ${}_7P_7 = 5\ 040$ **or** $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5\ 040$

b. Determine the number of different player arrangements that can be made for the picture if the tallest player must stand in the middle.

Possible Solutions:
$6! = 720$ **or** ${}_6P_6 = 720$ **or** $6 \cdot 5 \cdot 4 \cdot 1 \cdot 3 \cdot 2 \cdot 1 = 720$

24. Only six people, including Bill and Mary, have tickets entered to win the 2 prizes in a school draw, and each person has only one ticket. Once a ticket is drawn for a prize, it is not re-entered in the draw. What is the probability that Bill wins the first prize and Mary wins the second prize?

Possible Solutions:

$\frac{1}{6} \cdot \frac{1}{5} = \frac{1}{30}$ **or** $\frac{1}{{}_6P_2} = \frac{1}{30}$

Use the following information to answer numerical-response question 25.

A student is classifying the following contexts as either permutations or combinations.

Context A	Dialing a 10-digit telephone number with distinct digits
Context B	Choosing 5 people for a committee
Context C	Selecting 4 fruits to put in a salad
Context D	Entering a 3-digit phone passcode

Numerical Response

25. For each context, use a **1** to indicate that the context would be classified as a **permutation** and use a **2** to indicate that the context would be classified as a **combination**.

Context A __________

Context B __________

Context C __________

Context D __________

Solution:
1221

26. Triangles can be formed in an octagon by connecting any 3 of its vertices. Determine the number of different triangles that can be formed in an octagon.

Possible Solution:

$$\binom{n}{r} = \frac{n!}{(n-r)!r!}$$

$$\binom{8}{3} = \frac{8!}{(8-3)!3!}$$

$$= 56 \text{ triangles}$$

27. A fruit salad is to contain 1 green fruit, 2 different yellow fruits, and 3 different red fruits. The number of possible fruit salads that can be made from 2 different green fruits, 5 different yellow fruits, and 9 different red fruits is

A. 20 160
B. 8 008
***C.** 1 680
D. 90

Possible Solution:
$_2C_1 \bullet {}_5C_2 \bullet {}_9C_3 = 1\ 680$

28. Ralph knows that there are 15 distinguishable possibilities when 2 people are chosen to form a committee from a particular group of *n* people.

Describe any restrictions on the value of *n* in this context.

Possible Solution:
The value of *n* represents the number of people in the larger group. This must be a positive number, as mathematically you cannot have a negative number of people. Also, *n* must be greater than 2 because it is impossible to select two objects from a group smaller than what is needed.

Use the following information to answer numerical-response question 29.

A committee of 3 girls and 2 boys is to be chosen from a group of 9 girls and 7 boys. The total number of different committees that can be formed can be expressed in the form

$$_wC_x \bullet {}_yC_z$$

where $_wC_x$ represents the number of possible choices of girls for the committee and $_yC_z$ represents the number of possible choices of boys for the committee.

Numerical Response

29. The values of *w*, *x*, *y*, and *z* are _____, _____, _____, and _____, respectively.

Solution:
9372

30. In a group of 9 students, there are 4 females and 5 males.

a. How many different 4-member committees selected from this group have 2 females and 2 males?

Possible Solution:

$$\binom{4}{2} \cdot \binom{5}{2} = 60$$

SE

b. How many different 2-member or 3-member committees can be formed from this group of students?

Possible Solution:

$$\binom{9}{2} + \binom{9}{3} = 120$$

c. In the group of 9 students, 3 are in Grade ten, 3 are in Grade eleven and 3 are in Grade twelve. Determine the probability that 2 Grade ten students are chosen to be on a 2-member committee.

Possible Solutions:

$$\frac{{}_3C_2}{{}_9C_2} = \frac{1}{12} \quad \textbf{or} \quad \frac{3}{9} \cdot \frac{2}{8} = \frac{1}{12}$$

Note: This example is considered to be at the acceptable standard even though it is technically possible to consider ${}_6C_0$ in the numerator of the solution on the left. Since this is a step that students will not have to perform in order to arrive at the correct answer, it is considered to involve a single combination in the numerator.

SE

d. Determine the probability that a 4-member committee chosen at random from this group will consist of at least 3 females.

Possible Solution:

$$\frac{{}_4C_3 \cdot {}_5C_1}{{}_9C_4} + \frac{{}_4C_4}{{}_9C_4} = \frac{1}{6}$$

Standards for Relations and Functions

General Outcome

Develop algebraic and graphical reasoning through the study of relations.

General Notes:

- The emphasis of this topic is on the application of functions.
- Teachers may wish to use algebraic manipulations as well as regression models to explore the functions studied in this topic.
- Students should be able to relate graphical solutions to algebraic solutions.
- Students should be familiar with the terms *expression*, *equation*, *function*, *extraneous values*, and *non-permissible values*. Students should also be aware of solutions that need to be rejected because of the context of the problem.
- Technology may be useful in the exploration and understanding of the relationship between the parameters of an equation and the characteristics of the graph of the corresponding function.
- For specific outcomes 6, 7, and 8, if using a regression function for interpolation or extrapolation, students should use the non-rounded parameter values.
- When solving contextual problems, students must illustrate, explicitly or implicitly, the meaning of the variable quantities used.
- Specific outcomes 4, 5, and 6 are interrelated, and teachers may wish to explore them simultaneously. Although students may not be familiar with inverse functions, they can explore the symmetry between the graph of an exponential function and the graph of the corresponding logarithmic function. A detailed study of inverse functions is beyond the scope of Mathematics 30–2.
- Although there are many acceptable methods to express domain and range, students will only be expected to provide simplified set notation (e.g., $-3 < x < 8$). Unless otherwise specified, all domain and range statements are over the set of real numbers.

Specific Outcome 1
Determine equivalent forms of rational expressions (limited to numerators and denominators that are monomials and binomials). **[C, ME, R]**

Notes:

- Students should be aware of the need to state non-permissible values before simplifying a rational expression.
- Technology (T) has not been identified as one of the mathematical processes to be emphasized in completing this outcome. Although there may be some opportunities for students to use technology to investigate equivalent rational expressions, students are expected to meet this outcome without the use of technology.

(Refer to examples 1, 2, 3, 4, 5, 6, and 7.)

Specific Outcome 2
Perform operations on rational expressions (limited to numerators and denominators that are monomials and binomials). **[CN, ME, R]**

Notes:

- Students should connect their prior knowledge of operations on rational numbers to operations on rational expressions.
- When operations are performed on rational expressions, the resulting expression is not limited to a monomial or binomial.
- Operations should be limited to two rational expressions.
- Students should be aware of the need to state non-permissible values before performing operations on expressions.
- Technology (T) has not been identified as one of the mathematical processes to be emphasized in completing this outcome. Although there may be some opportunities for students to use technology to investigate operations on rational expressions, students are expected to meet this outcome without the use of technology.
- Students should be aware that $(x – a)$ and $(a – x)$ have equivalent binomial factors.

(Refer to examples 8, 9, and 10.)

Specific Outcome 3
Solve problems that involve rational equations (limited to numerators and denominators that are monomials and binomials). **[C, CN, PS, R]**

Notes:

- Students may need to review how to solve quadratic equations using a variety of methods, such as factoring and the quadratic formula.
- Students should be aware that the equations used to solve some contextual problems may have solutions that need to be rejected, extraneous solutions, or both.
- Students should be aware of the need to consider non-permissible values when solving problems that involve rational equations.
- Technology (T) has not been identified as one of the mathematical processes to be emphasized in completing this outcome. Although there may be some opportunities for students to use technology to investigate rational equations, students are expected to meet this outcome without the use of technology.

(Refer to examples 11, 12, 13, 14, and 15.)

Specific Outcome 4
Demonstrate an understanding of logarithms and the laws of logarithms. **[C, CN, ME, R] [ICT: C6–4.1]**

Notes:

- If no base is specified, it is assumed to be 10.
- Students may need to review the laws of exponents prior to beginning this outcome.
- The focus of this outcome is on developing students' conceptual knowledge of logarithms.
- The laws of logarithms can be verified using numerical values. Proofs of the laws of logarithms are beyond the scope of Mathematics 30–2.
- Natural logarithms will be used only for logarithmic regression (SO6).

(Refer to examples 16, 17, 18, 19, 20, 21, and 22.)

Specific Outcome 5

Solve problems that involve exponential equations. **[C, CN, PS, R, T] [ICT: C6–4.1, C6–4.3]**

Notes:

- Students may need to review laws of exponents and laws of logarithms prior to beginning this outcome.
- Students are expected to solve exponential equations both algebraically and graphically; however, complex equations such as $a^{(cx + d)} = b^{(ex + f)}$, where a and b cannot be written as powers with a common base, will only be solved graphically.
- Solving logarithmic equations, such as $\log x + \log 5 = 3$, is beyond the scope of Mathematics 30–2.

(Refer to examples 23 and 24.)

Specific Outcome 6

Represent data, using exponential and logarithmic functions, to solve problems.
[C, CN, PS, T, V] [ICT: C6–4.1, C6–4.3, C6–4.4]

Notes:

- The emphasis of this outcome should be on the application of exponents and logarithms. Applications could include decibel scale, Richter scale, growth and decay, and compound interest. However, students should not be required to determine payments for loans or mortgages.
- Teachers may wish to discuss the basic characteristics – such as intercepts, shape of graph, domain, and range – of exponential and logarithmic functions.
- When a contextual problem is solved, a discussion of the characteristics of an exponential or logarithmic function should focus on how the characteristics relate to the context of the problem.
- When providing an algebraic solution to a compound interest problem, students should be given a function that models the compounded value unless interest is compounded annually.
- Exponential regression or logarithmic regression can be used to solve problems; however, students will not be expected to distinguish between the appropriateness of an exponential regression model versus a logarithmic regression model for a given set of data.
- Students should be aware that e is an irrational number that is used in logarithmic regression. The conceptual understanding of the nature of e is beyond the scope of Mathematics 30–2.

(Refer to examples 25, 26, 27, 28, 29, 30, 31, 32, and 33.)

Specific Outcome 7
Represent data, using polynomial functions (of degree ≤ 3), to solve problems. **[C, CN, PS, T, V] [ICT: C6–4.1, C6–4.3, C6–4.4]**

Notes:

- The emphasis of this outcome should be on the application of polynomial functions.
- Teachers may wish to review the relationship between the roots of an equation, the zeros of the corresponding function, and the *x*-intercepts of the graph of the function.
- Teachers may wish to discuss the basic characteristics – such as intercepts, turning points, maximum, minimum, domain, and range – of each type of function (linear, quadratic, cubic).
- When a contextual problem is solved, a discussion of the characteristics of a polynomial function should focus on how the characteristics relate to the context of the problem.
- Contextual problems involving linear or quadratic functions may be solved algebraically or with technology. If regression is used, a minimum of five points should be used to determine an appropriate regression model. Contextual problems involving cubic functions should be solved with technology.
- Students are expected to explain why some solutions are not valid for the context of the problem.

(Refer to examples 34, 35, 36, 37, 38, 39, 40, 41, and 42.)

Specific Outcome 8
Represent data, using sinusoidal functions, to solve problems. **[C, CN, PS, T, V]**
[ICT: C6–4.1, C6–4.3, C6–4.4]

Notes:

- The emphasis of this outcome should be on the application of sinusoidal functions.
- It is not the intent of this outcome to do an in-depth study of sinusoidal functions, but to use sinusoidal models to show patterns from which inferences can be drawn. Applications will be limited to $y = \sin x$.
- Teachers may wish to discuss the basic characteristics of a sinusoidal function – such as intercepts, amplitude, period, maximum, minimum, median, midline, domain, and range.
- When a contextual problem is solved, a discussion of the characteristics of a sinusoidal function should focus on how the characteristics relate to the context of the problem.
- Technology may be used to solve contextual problems involving sinusoidal functions.
- Graphing calculators will model sinusoidal regression in the form $y = a \cdot \sin(bx + c) + d$, where x is measured in radians. This means that students will need to be familiar with radians as an additional unit of angle measure. However, it is not the intent for students to be able to convert from one unit to another.
- A minimum of five points over most of one period is normally sufficient to produce an adequate sinusoidal regression model. It is recommended that when entering values for sinusoidal regression into a graphing calculator, students begin with the lowest x-value.
- Students should connect the term *midline* to *median*.
- Students should be aware that when $c \neq 0$, a phase shift has occurred; but stating the magnitude or direction is beyond the scope of Mathematics 30–2.

(Refer to examples 43, 44, 45, 46, 47, 48, and 49.)

Acceptable Standard	Standard of Excellence
The student can	The student can also
• simplify a rational expression	
• determine whether two rational expressions are equivalent	• determine an equivalent rational expression, given a rational expression and the non-permissible values
• identify an error in the simplification of a rational expression	• identify and correct an error in the simplification of a rational expression
• state all non-permissible values for a given rational expression	
• explain why a non-permissible value exists	
• demonstrate by substitution that a value is non-permissible for a rational expression	
• determine the sum or difference of two rational expressions with the same denominators or different monomial denominators	
• determine the sum or difference of two rational expressions that have equivalent binomial factors in the denominator	• determine the sum or difference of two rational expressions that have non-equivalent binomial factors in the denominator
• determine the product or quotient of two rational expressions	
• state the non-permissible values of the sum, difference, product, or quotient of two rational expressions	• explain why, when rational expressions are divided, the divisor may produce additional non-permissible values for the quotient
• identify an error in operations with rational expressions	• identify and correct an error in operations with rational expressions
• algebraically solve a rational equation that simplifies to a linear equation and identify extraneous solutions	• algebraically solve a rational equation that simplifies to a quadratic equation and identify extraneous solutions
• identify an error in the solution to a rational equation	• identify and correct an error in the solution to a rational equation
• solve a contextual problem when given a rational equation that simplifies to a linear equation and identify extraneous solutions	• solve a contextual problem when given a rational equation that simplifies to a quadratic equation and identify extraneous solutions
	• determine a rational equation that models a given context

• explain why a solution of a rational equation might be rejected in a contextual problem	
• express a logarithm in exponential form and vice versa	
• evaluate $\log_a b$ when b is an integral power of a, without technology	
• evaluate $\log_a b$, with technology	
• apply the laws of logarithms to logarithms with numerical values	
• apply one of the product, quotient, or power laws to logarithms with variables	
• apply both product and quotient laws to logarithms with variables	• apply two or more logarithm laws to logarithms with variables where one of the laws is the power law
• estimate the solution of an exponential equation of the form $a = b^x$	
• solve exponential equations of the form $a = c \bullet b^x$, algebraically	• solve exponential equations of the form $a = c \bullet b^{(dx + e)}$, algebraically
• solve exponential equations of the form $a^{(cx + d)} = b^{(ex + f)}$, where a and b can be written as powers with a common base, algebraically	• solve exponential equations of the form $a = b^{(cx + d)}$, where a and b cannot be written as powers with a common base, algebraically
• solve any exponential equation graphically	
• identify an error in the solution to an exponential equation	• identify and correct an error in the solution to an exponential equation
• graph and analyze exponential or logarithmic functions, including intercepts, domain, and range, using technology	
• perform an exponential or logarithmic regression to obtain an appropriate function to solve problems	• explain how changes to the context of a problem affect an exponential function or its graph.
• determine an exponential function that could be used to model a contextual problem where a table of values or the graph is given	• determine an exponential function that could be used to model a contextual problem where a table of values or the graph is not given
• solve a contextual problem that can be modelled by an exponential function where the function, a table of values, or the graph is given	• solve a contextual problem that can be modelled by an exponential function where the function, a table of values, or the graph is not given

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

• solve a contextual problem that can be modelled by a logarithmic function where the function, a table of values, or the graph is given	
• graph data that can be modelled by an exponential or logarithmic function	
• graph and analyze polynomial functions, including intercepts, turning points, maximum, minimum, domain, and range, using technology	• identify the domain and range of a polynomial function, given a context
• perform a linear, quadratic, or cubic regression to obtain an appropriate function to solve problems	
• solve a contextual problem that can be modelled by a polynomial function where the function, a table of values, or the graph is given	• solve a contextual problem that can be modelled by a polynomial function where the function, a table of values, or the graph is not given • explain how changes to the context of a problem affect a polynomial function or its graph
• graph and analyze sinusoidal functions, including intercepts, amplitude, period, maximum, minimum, median, equation of midline, domain, and range	• identify the domain and range of a sinusoidal function, given a context
• perform a sinusoidal regression to obtain an appropriate function to solve problems	
• solve a contextual problem that can be modelled by a sinusoidal function where the function, a table of values, or the graph is given	• solve a contextual problem that can be modelled by a sinusoidal function where the equation, a table of values, or the graph is not given • explain how changes to the context of a problem affect a sinusoidal function or its graph
• identify an appropriate regression model, given data that represent a context	
• participate in and contribute toward the problem-solving process for problems that require the analysis of relations and functions studied in Mathematics 30–2	• complete the solutions to problems that require the analysis of relations and functions studied in Mathematics 30–2

Sample Questions

Students who achieve the acceptable standard should be able to answer all of the following questions, except for any part of a question labelled **SE**. Parts labelled **SE** are appropriate examples for students who achieve the standard of excellence.

Note: In the multiple-choice questions that follow, an asterisk (*) indicates the correct answer. Please be aware that the worked solutions shown are possible strategies; there may be other strategies that could be used.

1. An expression that is equivalent to $\frac{x^2 + x}{x}$, $x \neq 0$, is

A. x

B. x^2

***C.** $x + 1$

D. $x^2 + 1$

Numerical Response

2. The non-permissible value of x in the expression $\frac{2x+1}{3x-9}$ is ________.

Possible Solution:

$3x - 9 \neq 0$

$3x \neq 9$

$x \neq 3$

The non-permissible value of x is 3.

Use the following information to answer question 3.

Sanja and David both simplified the expression $\frac{x}{x^2 + x}$. Their work is shown below.

Sanja	**David**
$\frac{x}{x^2 + x}$	$\frac{x}{x^2 + x}$
$\frac{x}{x(x+1)}$	$\frac{{}^{1}\cancel{x}}{x^{\cancel{2}} + {}^{1}\cancel{x}}$
$\frac{1}{x+1}$	$\frac{1}{x+1}$

Sanja stated that the non-permissible values of x for the equivalent rational expressions are –1 and 0.

David stated that the non-permissible value of x for the equivalent rational expressions is –1.

3. Which student is correct? Justify your choice.

Possible Solution:
Sanja is correct. Both students correctly simplified the expression, but David made an error in stating the non-permissible values. Non-permissible values of x must be identical for equivalent rational expressions. The non-permissible values for the original expression were –1 and 0. They must be the same for the simplified expression.

4. Explain why the non-permissible value of x in the expression $\frac{3x}{x+2}$ is –2.

Possible Solution:
The non-permissible value for any rational expression is the value that, when substituted for the variable, makes the denominator equal 0 and, therefore, causes the rational expression to be undefined. In this case, if –2 is substituted for x, the denominator will be 0. This is why the non-permissible value of x is –2.

Use the following information to answer numerical-response question 5.

When the rational expression $\frac{2x+4}{x^2-4}$ is simplified, the equivalent expression can be written in the form $\frac{2}{\boldsymbol{A}}$, $x \neq \boldsymbol{B}$.

Expressions for ***A*** and ***B*** that would correctly complete the simplified form can be selected from the table below.

Code	Possibilities for *A*
1	$x - 2$
2	$x + 2$
3	x

Code	Possibilities for *B*
4	–2
5	0
6	–2, 2
7	–2, 0, 2

Numerical Response

5. In the equivalent expression $\frac{2}{\boldsymbol{A}}$, $x \neq \mathbf{B}$, the code for

A is ___________

B is ___________

Solution:
16

SE

6. A rational expression, where the non-permissible value of x is 1, has been simplified to x. Determine an equivalent rational expression for the simplified expression.

Possible Solution:
An equivalent expression could be $\frac{x(x-1)}{(x-1)}$ or $\frac{x^2-x}{x-1}$. This expression has a non-permissible value for x of 1.

Use the following information to answer numerical-response question 7.

The expression $\frac{AB}{C}$ can be simplified to $\frac{x+4}{x+3}$, $x \neq -3, 3$. Henry knows that one expression can be selected from each of the tables below to form the original expression.

Code	Possibilities for *A*
1	$(x-3)$
2	$(2x-6)$
3	$(3x-9)$

Code	Possibilities for *B*
4	$(2x+8)$
5	$(x+4)$
6	$\frac{1}{2}(x+4)$

Code	Possibilities for *C*
7	$(3x^2-27)$
8	$(2x^2-18)$
9	(x^2-9)

Numerical Response

SE **7.** One possible selection to form the original expression is $(x-3)$, $(x+4)$, and (x^2-9), so Henry records the code 159. To form another possible original expression, a code for

A is ________

B is ________

C is ________

(There is more than one correct answer.)

Possible Solutions:
258
148
269
357

Use the following information to answer numerical-response question 8.

The simplified product of $\frac{2n^4p}{3m} \cdot \frac{6m^6}{3n^2p^2}$, where $m \neq 0$ and $n \neq 0$ and $p \neq 0$, can be represented by

$$\frac{Am^Bn^C}{3p}$$

where A, B, and C represent single-digit whole numbers.

Numerical Response

8. In the simplified product $\frac{Am^Bn^C}{3p}$, the value of

A is __________

B is __________

C is __________

Solution:
452

9. Simplify the following.

a. $\frac{5}{3x^2} \bullet \frac{6x}{x+2}, x \neq -2, 0$

b. $\frac{x}{x+2} \bullet \frac{3x+6}{x-3}, x \neq -2, 3$

c. $\frac{x+3}{5x-1} \div \frac{2x+6}{4x}, x \neq -3, 0, \frac{1}{5}$

d. $\frac{x^2+3x}{x^2-16} \div \frac{x+3}{x+4}, x \neq -4, -3, 4$

Possible Solutions:

a.
$$\frac{5}{3x^2} \bullet \frac{6x}{x+2} = \frac{30x}{3x^2(x+2)}$$
$$= \frac{10}{x(x+2)}$$

b.
$$\frac{x}{x+2} \bullet \frac{3x+6}{x-3} = \frac{x(3)(x+2)}{(x+2)(x-3)}$$
$$= \frac{3x}{x-3}$$

c.
$$\frac{x+3}{5x-1} \div \frac{2x+6}{4x} = \frac{x+3}{5x-1} \bullet \frac{4x}{2(x+3)}$$
$$= \frac{2x}{5x-1}$$

d.
$$\frac{x^2+3x}{x^2-16} \div \frac{x+3}{x+4} = \frac{x(x+3)}{(x+4)(x-4)} \bullet \frac{x+4}{x+3}$$
$$= \frac{x}{x-4}$$

10. Simplify the following. State all non-permissible values.

a. $\frac{3}{5x}+\frac{7x}{4}$

b. $\frac{4x}{x-7}-\frac{5x+3}{x-7}$

c. $\frac{x+1}{3x^2-12x}+\frac{5}{2x-8}$

d. $\frac{x}{3-x}-\frac{3}{x-3}$

e. $\frac{2x}{x+3}-\frac{x-1}{x}$

SE **f.** $\frac{x}{4x^2-1}+\frac{3x}{2x^2+x}$

SE **g.** $\frac{x^2+3x}{x^2-4}+\frac{x^2+5x}{x+2}$

Possible Solutions:

a.
$$\begin{aligned}\frac{3}{5x}+\frac{7x}{4}&=\frac{12}{20x}+\frac{35x^2}{20x}\\&=\frac{12+35x^2}{20x},\ x\neq 0\end{aligned}$$

b.
$$\begin{aligned}\frac{4x}{x-7}-\frac{5x+3}{x-7}&=\frac{4x-5x-3}{x-7}\\&=\frac{-x-3}{x-7},\ x\neq 7\end{aligned}$$

c.
$$\begin{aligned}\frac{x+1}{3x^2-12x}+\frac{5}{2x-8}&=\frac{x+1}{3x(x-4)}+\frac{5}{2(x-4)}\\&=\frac{2(x+1)}{6x(x-4)}+\frac{5(3x)}{6x(x-4)}\\&=\frac{17x+2}{6x(x-4)},\ x\neq 0, 4\end{aligned}$$

d.
$$\begin{aligned}\frac{x}{3-x}-\frac{3}{x-3}&=\frac{x}{-(x-3)}-\frac{3}{x-3}\\&=\frac{-x-3}{x-3},\ x\neq 3\quad\textbf{or}\quad\frac{-(x+3)}{x-3},\ x\neq 3\quad\textbf{or}\quad\frac{x+3}{3-x},\ x\neq 3\end{aligned}$$

Possible Solutions:

e. $$\frac{2x}{x+3}-\frac{x-1}{x}=\frac{2x^2}{x(x+3)}-\frac{(x-1)(x+3)}{x(x+3)}$$
$$=\frac{2x^2}{x(x+3)}-\frac{x^2+2x-3}{x(x+3)}$$
$$=\frac{2x^2-x^2-2x+3}{x(x+3)}$$
$$=\frac{x^2-2x+3}{x(x+3)},\ \ x\neq 0,-3$$

f. $$\frac{x}{4x^2-1}+\frac{3x}{2x^2+x}=\frac{x}{(2x-1)(2x+1)}+\frac{3x}{x(2x+1)}$$
$$=\frac{x}{(2x-1)(2x+1)}+\frac{3\cancel{x}}{\cancel{x}(2x+1)}$$
$$=\frac{x}{(2x-1)(2x+1)}+\frac{3(2x-1)}{(2x-1)(2x+1)}$$
$$=\frac{x+6x-3}{(2x-1)(2x+1)}$$
$$=\frac{7x-3}{(2x-1)(2x+1)},\ \ x\neq\frac{-1}{2},0,\frac{1}{2}$$

g. $$\frac{x^2+3x}{x^2-4}+\frac{x^2+5x}{x+2}=\frac{x^2+3x}{(x+2)(x-2)}+\frac{(x^2+5x)(x-2)}{(x+2)(x-2)}$$
$$=\frac{x^2+3x+x^3+3x^2-10x}{(x+2)(x-2)}$$
$$=\frac{x^3+4x^2-7x}{(x+2)(x-2)}$$
$$=\frac{x(x^2+4x-7)}{(x+2)(x-2)},\ x\neq -2,2$$

Use the following information to answer question 11.

In parallel circuits, the total resistance of a circuit is determined by using the formula $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$, where R_T is the total resistance, R_1 is the resistance of one branch of the parallel circuit, and R_2 is the resistance of the other branch of the parallel circuit.

In a particular parallel circuit, one branch has 5 Ω more resistance than the other. This can be modelled by the equation

$$\frac{1}{R_T} = \frac{1}{x} + \frac{1}{x+5}$$

where x is the resistance of one branch of the parallel circuit, in ohms.

11. If the total resistance of this circuit, R_T, is 6 Ω, then the values for the resistance of the two branches of the circuit are

A. 3 Ω and 3 Ω

B. 3 Ω and 8 Ω

C. 5 Ω and 10 Ω

***D.** 10 Ω and 15 Ω

12. Determine the solution to each equation.

a. $\frac{5x-1}{4x+11} = \frac{3}{4}$

b. $\frac{3}{x} + \frac{5}{3} = 10$

c. $\frac{3x}{x-1} - \frac{4}{x} = 3$

SE **d.** $\frac{2x}{x+3} + \frac{x}{x-3} = \frac{18}{x^2-9}$

Possible Solutions:

a. $$\frac{5x-1}{4x+11}=\frac{3}{4}$$
$$4(5x-1)=3(4x+11)$$
$$20x-4=12x+33$$
$$8x=37$$
$$x=\frac{37}{8}$$

Since $\frac{37}{8}$ is not one of the non-permissible values, the solution is $x=\frac{37}{8}$.

b. $$\frac{3}{x}+\frac{5}{3}=10$$
$$9+5x=30x$$
$$9=25x$$
$$x=\frac{9}{25}$$

Since $\frac{9}{25}$ is not one of the non-permissible values, the solution is $x=\frac{9}{25}$.

c. $$\frac{3x}{x-1}-\frac{4}{x}=3$$
$$3x(x)-4(x-1)=3x(x-1)$$
$$3x^2-4x+4=3x^2-3x$$
$$-x=-4$$
$$x=4$$

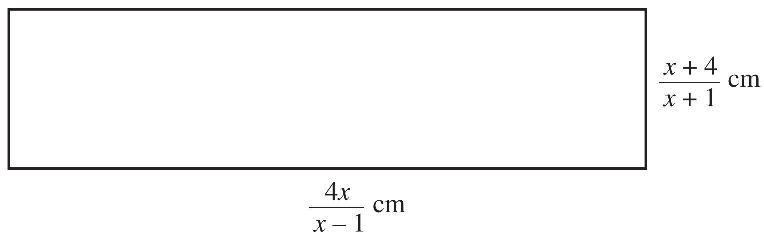
Since 4 is not one of the non-permissible values, the solution is $x = 4$.

d. $$\frac{2x}{x+3}+\frac{x}{x-3}=\frac{18}{x^2-9}$$
$$2x(x-3)+x(x+3)=18$$
$$2x^2-6x+x^2+3x=18$$
$$3x^2-3x-18=0$$
$$x^2-x-6=0$$
$$(x-3)(x+2)=0$$
$$x=3 \text{ or } x=-2$$

However, x cannot equal 3 because it makes one of the denominators in the rational equation equal to 0. Since $x = 3$ is rejected, the only solution is $x = –2$.

Use the following information to answer question 13.

The dimensions of a particular rectangle are represented by rational expressions, where $x > 1$, as shown in the diagram below.



SE **13.** If the area of the rectangle is 16 cm^2, determine the dimensions of the rectangle to the nearest centimetre.

Possible Solution:

$$\frac{4x}{x-1} \cdot \frac{x+4}{x+1} = 16$$

$$4x(x+4) = 16(x-1)(x+1)$$

$$4x^2 + 16x = 16x^2 - 16$$

$$0 = 12x^2 - 16x - 16$$

$$0 = 4(3x^2 - 4x - 4)$$

$$0 = 4(3x+2)(x-2)$$

$$x = \frac{-2}{3} \quad \text{or} \quad x = 2$$

Since $x > 1$, the solution to the equation is $x = 2$.

The length of the rectangle is $\frac{4 \cdot 2}{2-1} = 8$ cm.

The width of the rectangle is $\frac{2+4}{2+1} = 2$ cm.

Use the following information to answer question 14.

A student solved a rational equation using the steps shown below.

$$\frac{x}{x+1} - \frac{2}{x-1} = 2$$

Step 1 $x(x-1) - 2(x+1) = 2$

Step 2 $x^2 - x - 2x - 2 = 2$

Step 3 $x^2 - 3x - 4 = 0$

Step 4 $(x-4)(x+1) = 0$

Step 5 $x = -1$ or $x = 4$

SE **14. a.** Identify the errors made in the steps shown above.

Possible Solution:

Step 1: The student did not multiply the right-hand side of the equation by the common denominator.

Steps 2, 3, and 4: The student has carried the error from Step 1 through. (Note: If there had not been an error in Step 1, these steps would be correct.)

Step 5: Again, the student has carried the error from Step 1 through. However, the student has also made an error by not rejecting the extraneous solution $x = -1$.

SE **b.** Make the corrections necessary to obtain the solution to the equation.

Possible Solution:

Step 1 $x(x-1) - 2(x+1) = 2(x+1)(x-1)$

Step 2 $x^2 - x - 2x - 2 = 2x^2 - 2$

Step 3 $x^2 + 3x = 0$

Step 4 $x(x+3) = 0$

Step 5 $x = -3$ or $x = 0$

The non-permissible values are -1 and 1. Since -3 and 0 are not among the non-permissible values, the solution is $x = -3$ or $x = 0$.

Use the following information to answer question 15.

Elliott Nicholls currently holds the world record for the fastest text messaging while blindfolded. He was able to text 160 characters in a time that was 40 seconds less than the previous world record holder's time. Elliott's average rate of texting was 1.6 characters/second faster than the previous world record holder's average rate of texting. The chart below summarizes this information.

	Number of Characters	Time Taken (s)	Average Rate of Texting (characters/s)
Previous record holder	160	x	$\frac{160}{x}$
Elliott	160	$x - 40$	$\frac{160}{x-40}$

SE

15. a. Write an equation that models the relationship between the average rates of texting for Elliott and the previous world record holder.

Possible Solution:

$$\frac{160}{x-40} - \frac{160}{x} = 1.6$$

b. Describe the restrictions on the value of x in this context.

Possible Solution:
The value of x represents the time required to text 160 characters. Since Elliot beat the previous world record holder's time, x, by 40 s, the stated solution for x must be a positive value greater than 40 s.

c. The equation can be simplified to obtain $1.6x^2 - 64x - 6\,400 = 0$. Solve this equation. Express your solution to the nearest tenth.

Possible Solution:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{64 \pm \sqrt{(-64)^2 - 4(1.6)(-6\,400)}}{2(1.6)}$$
$$x = \frac{64 \pm \sqrt{45\,056}}{3.2}$$
$$x \approx 86.3 \quad \text{or} \quad x \approx -46.3$$

d. Do the solutions for x make sense in this context? Explain why or why not.

Possible Solution:
Of the two solutions, -46.3 s does not fit the context as it is a negative value for time, which has no logical meaning. Therefore, it must be rejected and the only solution is 86.3 s.

16. Write $4^2 = 16$ in logarithmic form.

Possible Solution:

$\log_4 16 = 2$

17. Evaluate $\log_2\left(\frac{1}{16}\right)$.

Solution:

$\log_2\left(\frac{1}{16}\right) = -4$

18. Evaluate $\log_a(a^5)$.

Solution:

$\log_a(a^5)$
$= 5\log_a(a)$
$= 5(1)$
$= 5$

19. Write each of the following logarithmic equations in exponential form.

a. $\log 100 = 2$

b. $\log_2 8 = 3$

c. $\log_a 5 = 2$

d. $\log_4(3x) = 9$

e. $2\log_5 m = 8$

f. $\log_3(x + 1) = 5$

Solutions:

a. $10^2 = 100$

b. $2^3 = 8$

c. $a^2 = 5$

d. $4^9 = 3x$

e. $5^8 = m^2$

f. $3^5 = x + 1$

20. Use the laws of logarithms to determine the value of each of the following expressions.

a. $\log_6 3 + \log_6 12$

b. $\log 520 - \log 52$

Possible Solutions:

a. $\log_6 3 + \log_6 12 = \log_6 36$
$= 2$

b. $\log 520 - \log 52 = \log 10$
$= 1$

21. Write the following expressions as single logarithms where $b > 1$.

a. $\log_b(2x) + \log_b(3x)$

Possible Solution:
$= \log_b(6x^2)$

SE **b.** $2 \log_b x - \log_b y$

Possible Solution:
$= \log_b(x^2) - \log_b y$
$= \log_b\left(\frac{x^2}{y}\right)$

22. Express each of the following expressions in a different logarithmic form.

a. $\log 6$

Possible Solutions:
$\log 2 + \log 3$
or
$\log 12 - \log 2$
or
$\frac{1}{2}\log 36$

SE **b.** $\log(xy^3)$

Possible Solution:
$= \log x + \log y^3$
$= \log x + 3\log y$

23. Solve algebraically.

a. $3 = 9^{2x}$

b. $2^{(x-1)} = 4^{(3x-1)}$

c. $10 = 3^{x}$

d. $2^{(5x)} = 13$

SE **e.** $2^{(2x-5)} = 3$

SE **f.** $3 \cdot 2^{(x-1)} = 24$

SE **g.** $3 \cdot 2^{(x-1)} = 70$

Possible Solutions:

a.
$$3 = 9^{2x}$$
$$3 = (3^2)^{2x}$$
$$3^1 = 3^{4x}$$
$$1 = 4x$$
$$x = \frac{1}{4}$$

b.
$$2^{(x-1)} = 4^{(3x-1)}$$
$$2^{(x-1)} = 2^{2(3x-1)}$$
$$2^{(x-1)} = 2^{(6x-2)}$$
$$x - 1 = 6x - 2$$
$$x = \frac{1}{5}$$

c.
$$10 = 3^{x}$$
$$x = \log_3 10$$
$$x = \frac{\log 10}{\log 3}$$
$$x \approx 2.1$$

or

$$10 = 3^{x}$$
$$\log 10 = \log 3^{x}$$
$$1 = x\log 3$$
$$x \approx 2.1$$

d.
$$2^{(5x)} = 13$$
$$5x = \log_2 13$$
$$x = \frac{\log_2 13}{5}$$
$$x \approx 0.74$$

or

$$2^{(5x)} = 13$$
$$\log 2^{(5x)} = \log 13$$
$$(5x)\log 2 = \log 13$$
$$5x = \frac{\log 13}{\log 2}$$
$$x \approx \frac{3.7}{5}$$
$$x \approx 0.74$$

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

e. $2^{(2x-5)} = 3$

$2x - 5 = \log_2 3$

$x = \log_2 3 + 5$

$x = \dfrac{\log_2 3 + 5}{2}$

$x \approx 3.3$

or

$2^{(2x-5)} = 3$

$\log 2^{(2x-5)} = \log 3$

$(2x-5)\log 2 = \log 3$

$2x - 5 = \dfrac{\log 3}{\log 2}$

$x = \dfrac{\log 3}{\log 2} + \dfrac{5}{2}$

$x \approx 3.3$

f. $3 \cdot 2^{(x-1)} = 24$

$2^{(x-1)} = 8$

$2^{(x-1)} = 2^3$

$x - 1 = 3$

$x = 4$

g. $3 \cdot 2^{(x-1)} = 70$

$2^{(x-1)} = \dfrac{70}{3}$

$x - 1 = \log_2\left(\dfrac{70}{3}\right)$

$x = \log_2\left(\dfrac{70}{3}\right) + 1$

$x \approx 5.54$

or

$3 \cdot 2^{(x-1)} = 70$

$2^{(x-1)} = \dfrac{70}{3}$

$\log 2^{(x-1)} = \log \dfrac{70}{3}$

$(x-1)\log 2 = \log \dfrac{70}{3}$

$(x-1) = \dfrac{\log \frac{70}{3}}{\log 2}$

$x = \dfrac{\log \frac{70}{3}}{\log 2} + 1$

$x \approx 5.54$

24. Describe how to determine the solution of $2^{(x-1)} = 3^{(x-2)}$ graphically.

Possible Solution:
Using a graphing calculator, graph $y_1 = 2^{(x-1)}$ and $y_2 = 3^{(x-2)}$, and use a window setting that shows both graphs and their intersection point. A possible window setting would be x:[–10, 10, 1], y:[–10, 10, 1]. Find the intersection point. The x-coordinate of this point is the solution to the equation.

Use the following information to answer question 25.

A car dealership observed and recorded the value of a particular car over time. It was discovered that the value of the car is decreasing at an average rate of 18%/a. The initial value of the car was $29 300.

SE

25. Which of the following exponential functions could be used to model the value of the car, $v(t)$, after t years?

A. $v(t) = 5\ 274(0.82)^t$
B. $v(t) = 5\ 274(1.18)^t$
***C.** $v(t) = 29\ 300(0.82)^t$
D. $v(t) = 29\ 300(1.18)^t$

Use the following information to answer question 26.

Sam deposits \$500 into a savings account that pays 2.4%/a, compounded annually. A function that models the growth of the deposit is

$$y = 500(1.024)^x$$

where y is the value of the investment, in dollars, and x is the number of years since the deposit was made.

26. **a.** Determine how long, to the nearest year, that it will take for the investment to be worth at least \$800.

Possible Solution:

$$\begin{aligned} y &= 500(1.024)^x \\ 800 &= 500(1.024)^x \\ 1.6 &= (1.024)^x \\ \log(1.6) &= \log(1.024)^x \\ \log(1.6) &= x \log(1.024) \\ \frac{\log(1.6)}{\log(1.024)} &= x \\ x &\approx 19.8 \end{aligned}$$

It will take 20 years for the investment to be worth at least \$800.

SE

b. Modify the exponential function to reflect an interest rate of 4%/a, compounded annually.

Possible Solution:

The value of b in the exponential function will change. The new function will be $y = 500(1.04)^x$.

Use the following information to answer question 27.

The intensity of an earthquake can be calculated using the formula

$$I = I_0(10)^M$$

where I represents the intensity of an earthquake, M is the magnitude of the earthquake on the Richter scale, and I_0 represents the intensity of an earthquake with a magnitude of 0.

27. Explain why an earthquake with a magnitude of 8.5 is approximately 40 times as intense as an earthquake with a magnitude of 6.9.

Possible Solution:

The earthquake with a magnitude of 8.5 has an intensity of $I = I_0(10)^{8.5}$, while the earthquake of magnitude 6.9 has an intensity of $I = I_0(10)^{6.9}$. When the ratio of these two intensities is calculated, $\frac{I_{8.5}}{I_{6.9}} = \frac{I_0(10)^{8.5}}{I_0(10)^{6.9}} = 10^{1.6}$, we see that the stronger earthquake is $10^{1.6}$ or approximately 40 times as intense as the weaker earthquake.

Use the following information to answer question 28.

The half-life of carbon-14 is approximately 5 730 years. As a sample of carbon-14 decays, the percentage of carbon-14 remaining, P, at any time during the process can be modelled by the function

$$P = 100\left(\frac{1}{2}\right)^{\left(\frac{t}{5\,730}\right)}$$

where t is the approximate age of the sample, in years.

28. To the nearest year, determine the approximate age of the carbon-14 when 33% of the original amount remains in the sample.

Possible Solution:

$$P = 100\left(\frac{1}{2}\right)^{\frac{t}{5\,730}}$$

$$33 = 100\left(\frac{1}{2}\right)^{\frac{t}{5\,730}}$$

$$0.33 = \left(\frac{1}{2}\right)^{\frac{t}{5\,730}}$$

$$\log_{\left(\frac{1}{2}\right)}(0.33) = \frac{t}{5\,730}$$

$$5\,730 \bullet \log_{\left(\frac{1}{2}\right)}(0.33) \approx t$$

$$t \approx 9164.918$$

The sample is approximately 9 165 years old.

Use the following information to answer question 29.

A researcher discovered mould growing in a Petri dish in her laboratory. When first observed, the mould covered only 3% of the dish's surface. Every 24 h, the surface area of the mould doubles in size, as shown in the table below.

Time (h)	Area covered (%)
0	3
24	6
48	
72	

29. a. Complete the table above and then write an exponential function to model the growth of the mould over time.

Possible Solution:

Time (h)	Area covered (%)
0	3
24	6
48	12
72	24

$y = 3(2)^x$, where x represents the number of 24-h periods since the first observation and y represents the percentage of area covered.

or

Using regression, $y = 3(1.029\ 302\ 237\ldots)^x$, where x represents the number of hours since the first observation and y represents the percentage of area covered.

b. Use your function from part (a) to determine the approximate length of time, to the nearest tenth of an hour, that it will take for the Petri dish to be completely covered with mould.

Possible Solutions:

$y = 3(2)^x$

$100 = 3(2)^x$

$\frac{100}{3} = 2^x$

$x = \log_2\left(\frac{100}{3}\right)$

$x = 5.058\ 893...$ 24-h periods

$5.058\ 893\ 689 \cdot 24 \approx 121.4$ h

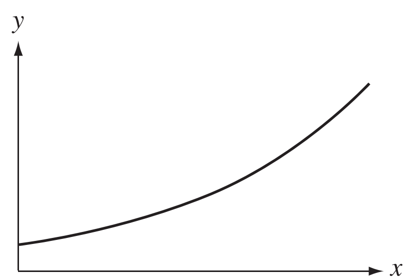
or

I used my calculator to graph $y_1 = 3(1.029\ 302\ 237)^x$ and $y_2 = 100$. Using the window setting x:[0,150,10], y:[0, 110, 10], I found the x-coordinate of the intersection point. It was $x = 121.4$.

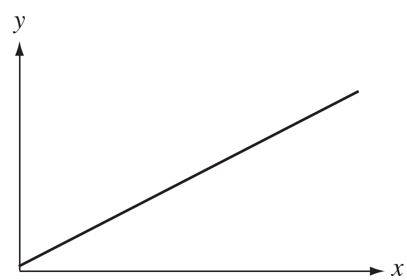
It will take approximately 121.4 h for the Petri dish to be completely covered with mould.

30. A painting was purchased in 2012 for \$10 000. If the painting appreciates in value at 5%/a, then which of the following graphs best models the value of this painting for the next 40 years, where x is the number of years since the painting was purchased and y is the value of the painting?

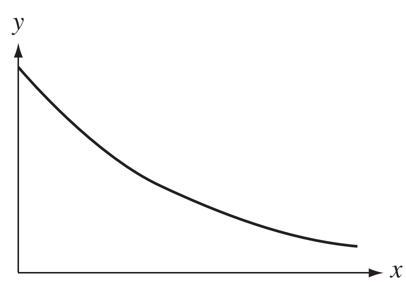
***A.**



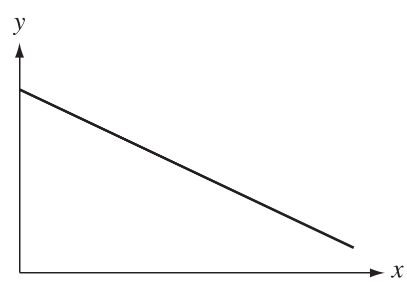
B.



C.



D.



Use the following information to answer numerical-response question 31.

The pH of a solution can be determined using the formula

$$pH = -\log_{10} [H_3O^+]$$

where $[H_3O^+]$ is the concentration of hydronium ions in the solution. The pH of a particular solution is 6.6.

Numerical Response

31. If the concentration of hydronium ions in the solution is doubled, the new pH of the solution, to the nearest tenth, will be __________.

Possible Solution:

$6.6 = -\log_{10} [H_3O^+]$

$10^{-6.6} = [H_3O^+]$

If the concentration is doubled, the new concentration will be $2 \cdot 10^{-6.6}$.

$pH = -\log_{10}(2 \cdot 10^{-6.6})$

$pH \approx 6.298\ 97$

$pH \approx 6.3$

Use the following information to answer question 32.

Corlene invested money in a GIC. The value of her investment for the first 3 years is shown in the table below.

Year	**Value of Investment**
0	\$1 000.00
1	\$1 020.00
2	\$1 040.40
3	\$1 061.21

32. **a.** To model the investment's growth and predict its future value, Corlene has chosen to use an exponential model. Discuss the effectiveness of her model.

Possible Solution:
Corlene made an effective choice of model because the growth rate of the investment is a constant percentage as time passes. This increasing function from a set starting point is characteristic of an exponential growth function.

b. Write an exponential function that Corlene could use to predict the future value of her investment.

Possible Solution:
Using an exponential function of the form $y = a \bullet b^x$, a possible function is $y = 1000(1.02)^x$, where x is the year and y is the value of the investment.

c. Explain what the numerical values in your function represent in the context of this problem.

Possible Solution:
In this function, a represents the initial value of the investment, which was \$1 000, and b represents the growth factor of the investment. In this case, the investment is growing at 2%/a, so the growth factor is 1.02.

SE

d. If Corlene invested in a GIC that paid 1.4%/a compounded annually, how would this affect the value of the investment over time?

Possible Solution:
If the interest rate was 1.4%/a compounded annually, the value of the investment over time would increase at a slower rate because this interest rate is lower than the other interest rate of 2%/a.

SE

e. If Corlene invested in a GIC that paid 1.4%/a compounded annually, how would this affect the function found in part b?

Possible Solution:
If the interest rate was 1.4%/a compounded annually, the function would change to $y = 1000(1.014)^x$, where x is the year and y is the value of the investment.

Use the following information to answer question 33.

When objects with different masses are compared without a scale, their masses must be different enough to be noticeable. The minimum amount of mass difference necessary to be noticeable is called the Minimum Perceivable Difference.

For heavier objects, the Minimum Perceivable Difference is larger. The Minimum Perceivable Differences for various masses are shown in the table below.

Mass (g)	**Minimum Perceivable Difference (g)**
100	5
200	10
400	15
800	20

These data can be modelled by a logarithmic regression function of the form

$$y = a + b \bullet \ln x$$

where x is the mass of the object, in grams, and y is the Minimum Perceivable Difference in mass, in grams.

33. **a.** Determine a logarithmic regression function that could be used to model these data. Round the values of a and b to the nearest tenth.

Possible Solution:
$y = -28.2 + 7.2 \ln x$

b. According to the logarithmic regression function, determine the Minimum Perceivable Difference, to the nearest gram, for an object with a mass of 2 100 g.

Possible Solution:
Let $x = 2\ 100$

$y = -28.219\ 280\ 95 + 7.213\ 475\ 204 \ln(2\ 100)$

$y = 26.961\ 587\ 11$

The Minimum Perceivable Difference in mass would be approximately 27 g.

34. Describe the graph of $f(x) = -(x + 1)^2(x - 2)$. Include the intercepts and turning points in your description.

Possible Solution:
The graph of this cubic function starts in the top left quadrant and ends in the bottom right quadrant. It has two distinct x-intercepts at $(-1, 0)$ and $(2, 0)$, and one y-intercept at $(0, 2)$. Viewed on a graphing calculator, this graph has a minimum turning point at $(-1, 0)$ and a maximum turning point at $(1, 4)$.

Use the following information to answer questions 35 and 37 and numerical-response question 36.

Water is being pumped into a 15-gallon tank. Once the volume of water in the tank reaches a certain amount, the tank begins to drain and continues draining until the water is completely gone. The volume of water in the tank can be modelled by the function

$$y = -2t^2 + 5t + 7$$

where y represents the volume of water in the tank in gallons and t represents the time in hours after noon on a particular day.

35. To determine the volume of water in the tank at noon, the characteristic of the graph of the function that should be analyzed is the

*__A.__ y-intercept
B. positive t-intercept
C. t-coordinate of the vertex
D. y-coordinate of the vertex

Numerical Response

36. The maximum volume of water in the tank, to the nearest tenth of a gallon, is __________ gal.

Possible Solutions:
Using a graphing calculator, the vertex is found to be approximately (1.25, 10.125). Therefore, the maximum volume of water in the tank is 10.1 gal.

or

$0 = -2t^2 + 5t + 7$

$0 = -1(2t - 7)(t + 1)$

$\therefore 2t - 7 = 0$ or $t + 1 = 0$

$t = 3.5 \qquad t = -1$

The vertex must be halfway between the two x-intercepts.
$\therefore\ t = 1.25$

When $x = 1.25$, $y = -2(1.25)^2 + 5(1.25) + 7$
$y = 10.125$

The vertex is (1.25, 10.125).
Therefore, the maximum volume of water in the tank is 10.1 gal.

SE **37.** Which of the following rows describes the **most appropriate** domain and range of the graph of the function in this context?

Row	**Domain**	**Range**
A.	$t \in R$	$y \leq 10.125$
B.	$t \in R$	$0 \leq y \leq 10.125$
C.	$0 \leq t \leq 3.5$	$y \leq 10.125$
***D.**	$0 \leq t \leq 3.5$	$0 \leq y \leq 10.125$

Use the following information to answer question 38.

The rate at which snow fell on a driveway on a particular day can be modelled by the function

$$y = -3x^2 + 6x$$

where y represents the rate of snowfall in cubic feet per hour, and x represents the time in hours.

38. Using the graph of the function to estimate the length of time that snow fell on this particular day, a student should determine the

A. y-intercept

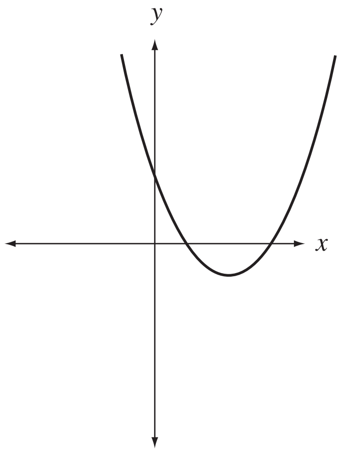
B. x-coordinate of the vertex

C. y-coordinate of the vertex

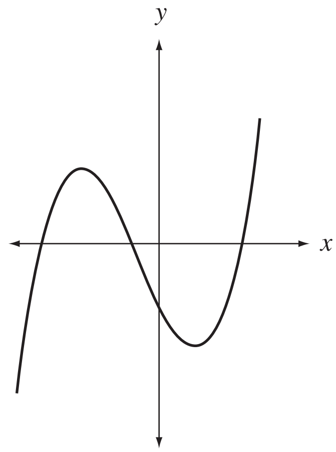
***D.** difference between the x-intercepts

39. Which of the following graphs is **most likely** the graph of a cubic function?

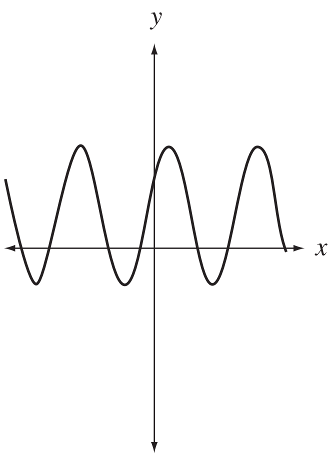
A.



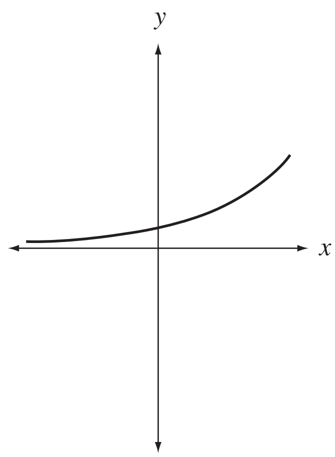
***B.**



C.



D.



Use the following information to answer question 40.

A hockey arena seats 1 600 people. The cost of a ticket is $10. At this price, every ticket is sold. To obtain more revenue, the arena management plans to increase the ticket price. A survey was conducted to estimate the potential revenue for different ticket prices, as shown below.

Ticket Price ($)	Potential Revenue ($)
10	16 000
15	19 500
20	20 300
25	14 750
30	5 500

The data above can be modelled by a quadratic regression function of the form

$$y = ax^2 + bx + c$$

where x is the ticket price, in dollars, and y is the potential revenue, in dollars.

40. Determine the ticket price that would maximize the revenue.

Possible Solution:
The quadratic regression function that models the given data is
$y = -91x^2 + 3\,125x - 6\,340$.

Using a window of x: [0, 40, 1], y: [0, 25 000, 1 000], the maximum of the function is at (17.17, 20 488.64). Therefore, the maximum potential revenue will be realized when the ticket price is approximately $17.17.

Use the following information to answer question 41.

A ball is launched vertically upward from a height of 5 ft, with an initial velocity of 96 ft/s. The height of the ball above the ground for the first 5 s is shown in the table below.

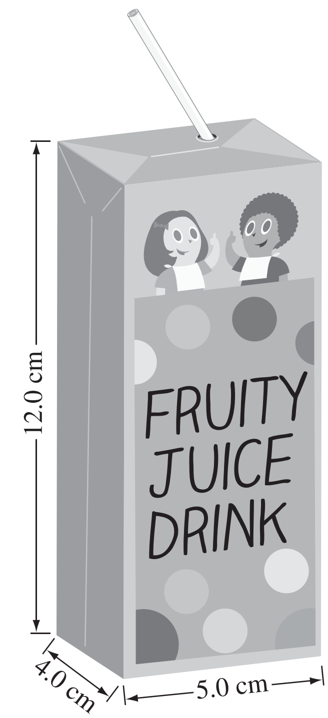
Time (s)	**Height (ft)**
0	5
1	86
2	134
3	150
4	134
5	86

41. These data could **most appropriately** be modelled using

A. linear regression
*__B.__ quadratic regression
C. sinusoidal regression
D. exponential regression

Use the following information to answer question 42.

A company wants to change the dimensions of a particular box, which will increase the volume. The box is currently 5.0 cm × 4.0 cm × 12.0 cm, as shown below.



To create the new box, the company will increase each dimension by the same amount. The volume of a box can be calculated by using the formula $V = l \cdot w \cdot h$.

SE

42. The company does not want the volume of the box to exceed 1 000 cm^3. Determine the largest amount by which each dimension of the box can be increased, to the nearest tenth of a centimetre.

Possible Solution:
Let x represent the amount of increase of each dimension.
$V = (5 + x)(4 + x)(12 + x)$
The volume must be at most 1 000 cm^3; therefore,
$1\,000 = (5 + x)(4 + x)(12 + x)$

This cubic equation can be solved using technology by sketching y_1=1 000 and $y_2 = (5 + x)(4 + x)(12 + x)$. Using the window x: [0, 10, 1], y: [0, 1 500, 100], the intersection point is (3.54, 1 000). The x-coordinate of the intersection point is the solution. Therefore, the largest amount that each dimension can be increased by is 3.5 cm to stay within the volume restriction.

Use the following information to answer numerical-response question 43.

The height of a pendulum, h, in inches, above a table top t seconds after the pendulum is released can be modelled by the sinusoidal function shown below.

$$h = 2\sin(3.14t + 1.57) + 5$$

Numerical Response

43. The height of the pendulum at the moment of release, to the nearest tenth of an inch, is __________ in.

Possible Solution:
The h-intercept represents the starting moment, so let $t = 0$.
$h = 2\sin(3.14(0) + 1.57) + 5$
$h = 6.999 \ldots$

The height of the pendulum at the moment of release is 7.0 in above the table top.

Use the following information to answer question 44 and numerical-response question 45.

The height above the ground of a rider on a Ferris wheel can be modelled by the sinusoidal function

$$h = 6\sin(1.05t - 1.57) + 8$$

where h is the height of the rider above the ground, in metres, and t is the time, in minutes, after the ride starts.

44. Based on the sinusoidal function, the maximum height of the rider above the ground is

- **A.** 2 m
- **B.** 6 m
- **C.** 8 m
- ***D.** 14 m

Numerical Response

45. When the rider is at least 11.5 m above the ground, she can see the rodeo grounds. During each rotation of the Ferris wheel, the length of time that the rider can see the rodeo grounds, to the nearest tenth of a minute, is __________ min.

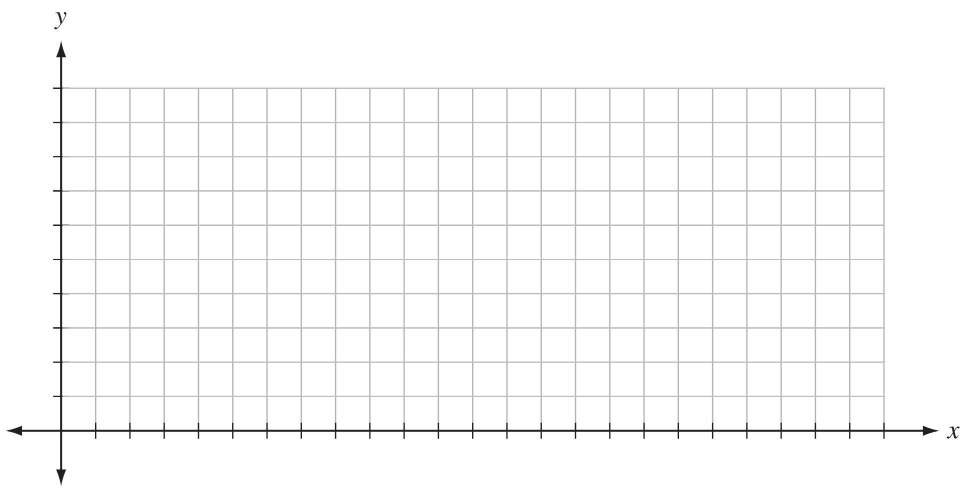
Possible Solution:
By sketching $y_1 = 11.5$ and $y_2 = 6\sin(1.05x - 1.57) + 8$ using a window of x:[0, 7, 1], y:[0, 15, 1] and finding the intersection points of the graphs, it can be determined that the rider can see the rodeo grounds between approximately 2.09 min and 3.89 min on the first rotation. This means the rider sees the rodeo grounds for approximately 1.8 min on each rotation.

Use the following information to answer question 46.

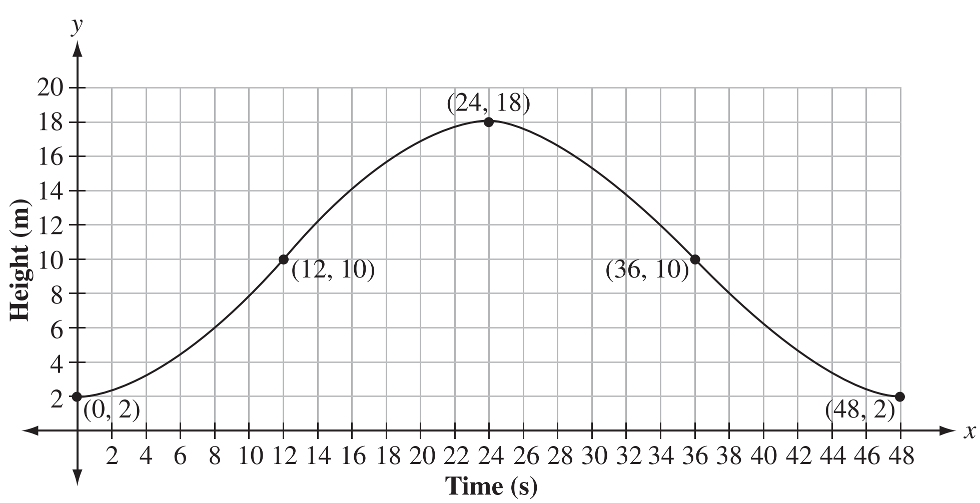
A Ferris wheel has a radius of 8 m and its centre is 10 m above the ground. A rider gets on a chair of the Ferris wheel at its lowest point and completes one full revolution in 48 s.

SE

46. **a.** Sketch a graph on the grid below to show the height of the rider above the ground, *y*, over time, *x*, for the first 48 s. Label key points on the graph.



Possible Solution:



b. State the amplitude, period, and equation of the midline for the function sketched in part a, on the previous page.

Possible Solution:
The amplitude represents the radius of the Ferris wheel, which is 8 m, and the period is 48 s. The midline represents the height of the centre of the Ferris wheel above the ground. The equation of the midline is $y = 10$.

c. Determine a function of the form $y = a \bullet \sin(bx - 1.57) + d$, where y represents the height of a rider above the ground and x represents the time after the ride has started, that could be used to model the height above the ground of a rider on the Ferris wheel described above.

Possible Solution:
The value of b can be determined by the formula $\frac{2\pi}{b}$ = period. $\frac{2\pi}{b} = 48$, so $b \approx 0.13$.
The function that models the height of a rider on this Ferris wheel is
$y = 8\ \sin(0.13x - 1.57) + 10$.

Use the following information to answer question 47.

The average daily high temperature of Montreal, in °F, for each month of the year is shown in the table below. (January = 1, February = 2, etc.)

Month	Average Daily High Temperature in °F
1	22
2	25
3	36
4	52
5	66
6	75

Month	Average Daily High Temperature in °F
7	80
8	77
9	67
10	51
11	41
12	28

47. **a.** Write a sinusoidal regression function of the form $y = a \bullet \sin(bx + c) + d$, where x is the month number and y is the average daily high temperature, that could be used to model these data. Round the values of a, b, c, and d to the nearest hundredth.

Possible Solution:
$y = 29.08 \sin(0.51x - 2.02) + 50.77$

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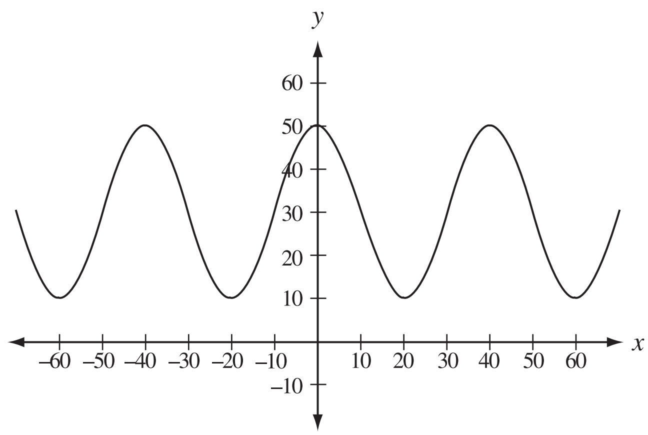
b. If scientists predict that the average daily high temperature in °F will increase by 1.2 °F each month, what characteristics of the graph of the sinusoidal regression function would change?

Possible Solution:
If the y-coordinate of every point on the graph of the sinusoidal function was increased by 1.2 °F, then there would be no change in the amplitude, period, and phase shift of the function. The median value would increase by 1.2 units, as would the maximum and minimum values, and the range of the graph. The domain would remain the same.

Use the following information to answer question 48.

The graph of a sinusoidal function is shown below.



Possible values for the amplitude and median of the sinusoidal function are 10, 20, 30, 40, and 50.

48. Record the values of the amplitude and the median of the sinusoidal function above in the blanks.

Value:	________	________
Characteristic:	**Amplitude**	**Median**

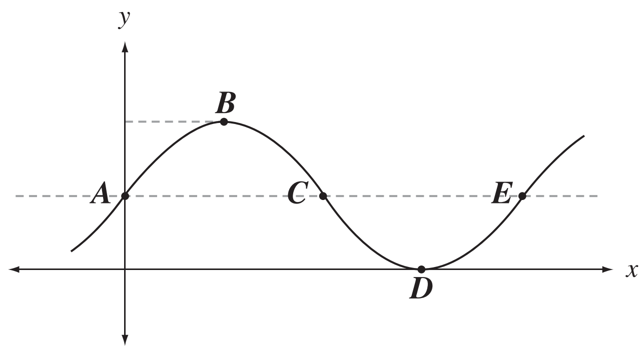
Solution:

Value:	20	30
Characteristic:	**Amplitude**	**Median**

Note: This question could be adapted to be an interactive item on a digital test.

Use the following information to answer question 49.

The graph of a sinusoidal function is shown below. The points ***A***, ***B***, ***C***, ***D***, and ***E*** are labelled. Points ***A***, ***C***, and ***E*** lie on the midline of the function.



49. a. Mary says that in order to find the period of the function, she would need to know the coordinates of points *A* and *E*. Bill says that he could find the period using the coordinates of *B* and *D*. Both Mary and Bill are correct. Explain why.

Possible Solution:
Mary is correct because the horizontal distance between *A* and *E* would be the period, as it is the length of time it takes the function to complete one cycle. Bill is also correct, as the horizontal distance between *B* and *D* represents half the period. He would need to remember to double this result to determine the period.

b. Select all points that represent the *x*-intercepts of the function.

Solution:
D is the only *x*-intercept visible on the graph above.

c. Select all points that represent the minimum value of the function.

Solution:
D is the only minimum point visible on the graph above.

d. Select two points that could be used to determine the amplitude of the function. Describe a process that could be used to determine the amplitude using the two selected points.

Possible Solution:
Select *B* and *C* and find the vertical distance between them by subtracting the *y*-coordinate of Point C from the *y*-coordinate of Point B. This distance would represent the amplitude.

Mathematics 30–2 Formula Sheet

Relations and Functions

Graphing Calculator Window Format

x: $[x_{\min}, x_{\max}, x_{\text{scl}}]$

y: $[y_{\min}, y_{\max}, y_{\text{scl}}]$

Exponents and Logarithms

$y = a^x \leftrightarrow x = \log_a y$

$\log_b c = \dfrac{\log_a c}{\log_a b}$

Laws of Logarithms

$\log_b(M \bullet N) = \log_b M + \log_b N$

$\log_b\left(\dfrac{M}{N}\right) = \log_b M - \log_b N$

$\log_b(M^n) = n \log_b M$

Exponential functions

$y = a \bullet b^x$

Logarithmic functions

$y = a + b \bullet \ln x$

Sinusoidal functions

$y = a \bullet \sin(bx + c) + d$

$\text{Period} = \dfrac{2\pi}{b}$

Quadratic equations

For $ax^2 + bx + c = 0$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Probability

$n! = n(n-1)(n-2)\ldots 3 \bullet 2 \bullet 1,$
where $n \in N$ and $0! = 1$

$_nP_r = \dfrac{n!}{(n-r)!}$

$_nC_r = \dfrac{n!}{(n-r)!r!}$

$_nC_r = \dbinom{n}{r}$

$P(A \cup B) = P(A) + P(B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cap B) = P(A) \bullet P(B)$

$P(A \cap B) = P(A) \bullet P(B \mid A)$

Logical Reasoning

A' Complement

$\varnothing$ Empty set

$\cap$ Intersection

$\subset$ Subset

$\cup$ Union

Appendix: Research Project

General Outcome

Develop an appreciation of the role of mathematics in society.

Specific Outcome 1

Research and give a presentation on a current event or an area of interest that involves mathematics. **[C, CN, ME, PS, R, T, V] [ICT: C1–4.2, C1–4.4, C2–4.1, C3–4.1, C3–4.2, C7–4.2, F2–4.7, P2–4.1]**

Notes:

- The intent of this project is to generate interest in mathematics and appreciate how mathematics is used in the real world.
- Regularly directing students' attention to news stories and current events that have a mathematical connection will allow students to have discussions about real-life applications and connections to mathematics. When students begin their own research projects, they will then have some knowledge to help them generate ideas to explore.
- Teachers should discuss how to critically evaluate information sources with students.
- Teachers should emphasize the necessity of citing information sources.
- Projects can be presented in various formats—multimedia, poster, oral presentation, written, social media site, etc.
- Teachers may wish to collaborate with their English and Social Studies colleagues when developing research project rubrics.
- For tips on developing rubrics, teachers may wish to refer to Assessment Strategies and Tools: Checklists, Rating Scales and Rubrics.
- Teachers may also wish to use the following sample rubrics and scoring guides when developing research project rubrics:
  - Science 30 Guides (embedded in each project)
  - Interpreting Evidence of Learning

Students should be encouraged to select a research topic that is of interest to them. However, if they are having trouble getting started, some possible broad topics are listed below.

- Write a position paper arguing for or against mathematics education in schools.
- Relate mathematics to a construction or design project.
- Apply mathematics to personal finance.
- Create a strategy game using mathematics.
- Explore the golden rectangle.
- Explore the Fibonacci sequence.
- Explore a connection between mathematics and art, music, or architecture.
- Conduct a study of bacterial growth, population growth, or the spread of a disease.
- Explore a connection between mathematics and the environment.
- Explore a connection between mathematics and personal health or fitness.
- Investigate the connection between mathematics and medications in the body.
- Investigate the connection between video games and mathematics.
- Research and report on a famous mathematician.
- Develop a study guide for Mathematics 30–2.
- Describe strategies for winning a game of choice.
- Create a poster of mathematical irregularities (e.g., $0! = 1$).
- Analyze cases of mathematical illiteracy in the media.