Assessment Standards & Exemplars

Mathematics 30–1

Alberta Provincial Diploma Examinations

This document was written primarily for:

Students	✓
Teachers	✓ of Mathematics 30–1
Administrators	✓
Parents	
General Audience	
Others	

Distribution: This document is posted on the Alberta Education website.

Contents

Please note that if you cannot access one of the direct website links referred to in this document, you can find Diploma Examination-related materials on the Alberta Education website.

Introduction

This resource is designed to support the implementation of the *Alberta Mathematics Grades 10–12 Program of Studies,* which can be found on the Alberta Education website. Teachers are strongly encouraged to consult the program of studies for details about the philosophy of the program.

The examples shown in this document were chosen to illustrate the intent of particular outcomes of Mathematics 30–1 but will not necessarily be assessed on a diploma examination in the manner shown. The examples provided are neither prescriptive nor exhaustive; they are intended to provide a profile of both acceptable achievement and excellent achievement. All examples were developed and validated by classroom teachers of mathematics but have not been validated with students. For examples of questions that have been used on previous diploma exams, please go to the Quest A+ website.

To meet the outcomes of Mathematics 30–1, students will need access to an approved graphing calculator. In most classrooms, students will use a graphing calculator daily. Refer to the calculator policy in the *General Information Bulletin* or go to the Alberta Education website for a list of approved graphing calculators. Information about the diploma examinations each year can also be found in the *Mathematics 30–1 Information Bulletin*.

The Provincial Assessment Sector would like to recognize and thank the many teachers throughout the province who helped to prepare this document. We would also like to thank the Programs of Study and Resources Sector and the French Language Education Services Branch for their input and assistance in reviewing these standards.

If you have comments or questions regarding this document, please contact Delcy Rolheiser by email at Delcy.Rolheiser@gov.ab.ca, by phone at 780-415-6181, or by fax at 780-422-4454.

Standards for Mathematics 30−1

The word *and* used in the standards implies that both ideas should be addressed at the same time or in the same question.

The assessment standards for Mathematics 30–1 include an acceptable and an excellent level of performance.

Acceptable Standard

Students who attain the acceptable standard but not the standard of excellence will receive a final course mark between 50% and 79% inclusive. Typically, these students have a proficient level of number sense, algebra skills, mathematical literacy, reading, comprehension, and reasoning. They have gained new skills and a basic knowledge of the concepts and procedures relative to the general and specific outcomes defined for Mathematics 30–1 in the program of studies. They demonstrate mathematical skills as well as conceptual understanding and can apply their knowledge to familiar problem contexts.

Standard of Excellence

Students who attain the standard of excellence will receive a final course mark of 80% or higher. Typically, these students have an advanced level of number sense, algebra skills, mathematical literacy, reading, comprehension, and reasoning. They have gained a breadth and depth of understanding regarding the concepts and procedures, as well as the ability to apply this knowledge and conceptual understanding to a broad range of familiar and unfamiliar problem contexts.

General Notes

- The seven mathematical processes [C, CN, ME, PS, R, T, V] should be used and integrated throughout as indicated in the specific outcomes.
- If technology [T] has not been specifically listed as a mathematical process for an outcome, teachers may, at their discretion, use it to assist students in exploring patterns and relationships when learning a concept. However, it is expected that technology will not be considered when assessing students' understandings of such outcomes.
- Most high school mathematics resources in North America use the letter *I* to represent the set of integers; however, students may encounter resources, especially at the post-secondary level, that use the letter *Z* to represent the set of integers. Both are correct.
- Students need to be familiar with the use of the letter *N* to represent the set of natural numbers and the use of the letter *W* to represent the set of whole numbers.
- For students transitioning between French and English instruction in mathematics, teachers can reference the mathematical terminology of both languages using the Mathematical Lexicon English-French provided on the Alberta Education website under Support Documents.

Performance Level Descriptors for Mathematics 30–1

These **Performance Level Descriptors** list attributes and abilities of students who attain each standard level.

Students who **just meet** *the acceptable standard are able to:*

- demonstrate knowledge of a particular concept through either visual or numerical representations
- demonstrate proficiency in number sense, algebra skills, mathematical literacy, reading, comprehension, and reasoning
- solve knowledge-based questions by making basic connections (ask "how") and focus on algorithms/procedural knowledge and basic conceptual knowledge
- comprehend the characteristics of a relation or function when a graph is shown or the equation is given
- solve familiar problems
- use and state a strategy to solve problems
- use an algebraic process to solve problems, verify solutions, or identify errors
- use technology to solve problems
- demonstrate mathematical processes that lead toward a complete solution when solving problems

Students who **just meet** *the standard of excellence are also able to:*

- demonstrate transferability of knowledge, making connections between topics, including abstract representations
- demonstrate an advanced level of number sense, algebra skills, mathematical literacy, reading, comprehension, and reasoning
- solve knowledge-based questions by making advanced connections (ask "why") and focus on conceptual/problem-solving knowledge
- apply the characteristics and/or changes in the context of a relation or function to create a graph or equation
- solve unique and unfamiliar problems
- use, interpret, and compare multiple strategies to solve problems
- use multiple algebraic processes to solve problems, verify solutions, and employ various methods of error identification
- use technology proficiently to solve problems and verify solutions
- demonstrate mathematical processes that result in a complete or correct solution when solving problems

Relations and Functions

General Outcome

Develop algebraic and graphical reasoning through the study of relations.

General Notes:

- Transformations covered in specific outcomes 2 to 5 include the algebraic base functions: $y = \frac{1}{x}$, $y = x$, $y = x^2$, $y = x^3$, $y = \sqrt{x}$, $y = |x|$, $y = b^x$, and $y = \log_b x$, with appropriate restrictions, as well as other graphical representations of functions.
- Stretches and reflections are performed prior to translations unless otherwise stated.
- Analyzing a transformation for specific outcomes 2 to 5 includes, but is not limited to, determining and describing the effects of a transformation on the domain, range, intercepts, invariant points, and key points.
- Analyzing graphs for specific outcomes 9 and 12 to 14 includes, but is not limited to, determining and describing the domain, range, intercepts, invariant points, and key points.
- The term *key points* on a relation or function may include vertices, endpoints, maximum and minimum points, etc.
- Technology [T] is not one of the mathematical processes listed for specific outcomes 2 to 7, 10, and 11. While technology can be used to discover and investigate the concepts, students are expected to meet these outcomes without the use of technology.
- Students should understand mapping notation for transformations.
- Students should be able to express any domain or range in both interval notation and set builder notation. For example:

Set Builder Notation	Interval Notation
Domain: $\{x \mid -1 \le x, x \in R\}$	Domain: $[-1, \infty)$
Range: $\{y \mid y \in R\}$	Range: $(-\infty, \infty)$

Be aware that both interval notation and set builder notation are different in French as detailed in the French version of this document.

Specific Outcomes

Specific Outcome 1

Demonstrate an understanding of operations on, and compositions of, functions. **[CN, R, T, V]** **[ICT: C6–4.1]**

Notes:

- The original functions used in operations and compositions should be limited to linear, quadratic, cubic, radical (one linear radicand), rational (monomial, binomial), absolute value (first degree only), exponential, logarithmic, and piecewise functions.
- For composition of functions, students should be familiar with the following notation:

$$\left(f \circ g\right)(x) = f\left(g(x)\right)$$

- For operations on functions, students should be familiar with the following notation:

$$(f + g)(x) = f(x) + g(x)$$

$$(f - g)(x) = f(x) - g(x)$$

$$(f \bullet g)(x) = f(x)g(x)$$

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

- Graphing technology may be used to analyze functions that result from operations or compositions that are beyond the scope of this course.
- The intent of this outcome is to focus on the conceptual understanding of operations and compositions rather than on lengthy algebraic processes.

(See examples 1-9.)

Specific Outcome 2

Demonstrate an understanding of the effects of horizontal and vertical translations on the graphs of functions and their related equations. **[C, CN, R, V]**

(See examples 10-12 and 55.)

Specific Outcome 3

Demonstrate an understanding of the effects of horizontal and vertical stretches on the graphs of functions and their related equations. **[C, CN, R, V]**

Note:

- Stretches about a line parallel to the x- or y-axis are beyond the scope of this course.

(See examples 13-16.)

Specific Outcome 4

Apply translations and stretches to the graphs and equations of functions. **[C, CN, R, V]**

(See examples 17, 18, and 20.)

Specific Outcome 5

Demonstrate an understanding of the effects of reflections on the graphs of functions and their related equations, including reflections through the:

- x-axis
- y-axis
- line $y = x$ **[C, CN, R, V]**

Notes:

- Reflections about a line parallel to the x-axis or y-axis are beyond the scope of this course.
- Reflections in the line $y = x$ should not be combined with any other transformations.

(See examples 15, 16, and 18-22.)

Specific Outcome 6

Demonstrate an understanding of inverses of relations. **[C, CN, R, V]**

Notes:

- Students should be familiar with the notation $x = f(y)$.
- The notation $y = f^{-1}(x)$ should only be used if the inverse is also a function.
- When discussing the **equations** of inverse relations, the focus should primarily be on linear, quadratic, exponential, or logarithmic functions.

- When exploring inverse relations **graphically**, teachers may choose to explore various relations, such as polynomial, piecewise, radical, exponential, logarithmic, and absolute values.
- Students should be able to restrict the domain on the original function to obtain an inverse that is also a function.

(See examples 21, 23, and 24.)

Specific Outcome 7

Demonstrate an understanding of logarithms. **[CN, ME, R]**

(See examples 4, 25, and 26.)

Specific Outcome 8

Demonstrate an understanding of the product, quotient, and power laws of logarithms.
[C, CN, ME, R, T] [ICT: C6–4.1]

Note:

- Change of base identity can be taught as a strategy for evaluating logarithms.

(See examples 27-30.)

Specific Outcome 9

Graph and analyze exponential and logarithmic functions. **[C, CN, T, V]**
[ICT: C6–4.3, C6–4.4, F1–4.2]

Notes:

- When graphing $y = a(b)^{x-c} + d$, the value of b will be restricted to $b > 0, b \neq 1$.
- When graphing $y = a\log_b(x - c) + d$, the value of b will be restricted to $b > 1$.
- Natural logarithms and base e are beyond the scope of this course.

(See examples 5, 22, and 31-34.)

Specific Outcome 10

Solve problems that involve exponential and logarithmic equations. **[C, CN, PS, R]**

Notes:

- Logarithmic equations should be restricted to same bases.

- Formulas will be given for any problems involving logarithmic scales such as decibels, earthquake intensity, and pH.
- Formulas will be given unless the context fits the form $y = ab^{\frac{t}{p}}$, where y is the final amount, a is the initial amount, b is the growth/decay factor, t is the total time, and p is the period.
- Compound interest involves an application of the formula $y = ab^{\frac{t}{p}}$. Students should be familiar with terms used for compound periods.

(See examples 35-40.)

Specific Outcome 11

Demonstrate an understanding of factoring polynomials of degree greater than 2 (limited to polynomials of degree ≤ 5 with integral coefficients). **[C, CN, ME]**

Note:

- The rational zero theorem is beyond the scope of this outcome; i.e., there will be at most two linear factors with a leading coefficient not equal to 1.

(See examples 41-45.)

Specific Outcome 12

Graph and analyze polynomial functions (limited to polynomial functions of degree ≤ 5).
[C, CN, T, V] [ICT: C6–4.3, C6–4.4]

Notes:

- Students must understand the relationship between zeros of a function, roots of an equation, *x*-intercepts of a graph, and factors of a polynomial.
- Analyzing a polynomial function graphically includes: leading coefficient, maximum and minimum points, domain, range, *x*- and *y*-intercepts, zeros, multiplicity, odd and even degrees, and end behaviour.
- Students should be able to identify when no real roots exist, but the calculation of them is beyond the scope of this outcome.
- The terms *maximum point* and *minimum point* refer to the absolute maximum and absolute minimum points, respectively.

(See examples 5, 44, and 46-50.)

Specific Outcome 13

Graph and analyze radical functions (limited to functions involving one radical). **[CN, R, T, V] [ICT: C6–4.1, C6–4.3]**

Notes:

- Radical functions will be limited to square roots.
- This specific outcome includes sketching and analyzing the transformation of $y = f(x)$ to $y = \sqrt{f(x)}$. The function $y = f(x)$ may be a linear, quadratic, or piecewise function.

(See examples 5, 20, and 51-53.)

Specific Outcome 14

Graph and analyze rational functions (limited to numerators and denominators that are monomials, binomials, or trinomials). **[CN, R, T, V] [ICT: C6–4.1, C6–4.3, C6–4.4]**

Notes:

- Oblique or slant asymptotes are not a part of this outcome.
- Numerators and denominators should be limited to degree two or less.
- This specific outcome does NOT include the transformation of $y = f(x)$ to $y = \frac{1}{f(x)}$.
- Horizontal asymptotes are limited to transformations of $y = \frac{1}{x}$ and $y = \frac{1}{x^2}$.

(See examples 5, 6, and 54-57.)

Specific Outcomes	Acceptable Standard The student can	Standard of Excellence The student can also
SO 1	• given their equations, sketch the graph of a function that is the sum, difference, product, quotient, or composition of two functions	
	• given their graphs, sketch the graph of a function that is the sum, difference, or product of two functions	• given their graphs, sketch the graph of a function that is the quotient of two functions
	• write a function, $h(x)$, as – the sum or difference of two or more functions – the product or quotient of two functions – a single composition E.g., $h(x) = f\big(f(x)\big)$ $h(x) = f\big(g(x)\big)$	• write a function, $h(x)$, as – the product or quotient of three functions – the composition of functions involving two compositions E.g., $j(x) = (f \circ g \circ h)(x)$ $h(x) = f\Big(g\big(f(x)\big)\Big)$
		• write a function, $h(x)$, combining two or more functions through operations on and/or compositions of functions, limited to two operations E.g., $h(x) = g(x) + f\big(g(x)\big)$ $h(x) = (f \bullet g)(x) - k(x)$
	• determine the domain and range of a function that results from the operation of two functions (i.e., sum, difference, product, or quotient)	• determine the domain of a function that is the composition of two functions
	• determine the value of operations or compositions of functions at a point E.g., $h(a) = (f \bullet g)(a)$ $h(a) = f\big(f(a)\big)$ $h(a) = (f \circ g)(a)$ $h(a) = f\Big(g\big(k(a)\big)\Big)$ $h(a) = g(a) + f\big(g(a)\big)$	

SO 2 SO 3 SO 4 SO 5	• perform, analyze, and describe graphically or algebraically – a combination of transformations involving stretches and/or translations – a combination of transformations involving reflections and/or translations – a combination of transformations involving reflections and/or stretches – a horizontal stretch and/or reflection in the *y*-axis and a translation where the parameter *b* is removed through factoring given the function in equation or graphical form or mapping notation • perform, analyze, and describe a reflection in the line $y = x$, given the function or relation in graphical form	• perform, analyze, and describe graphically or algebraically – a horizontal stretch and/or reflection in the *y*-axis and translation where the parameter *b* is not removed through factoring – a combination of transformations involving at least a reflection, a stretch, and a translation given the function in equation or graphical form or mapping notation
SO 6 SO 9	• determine the equation of the inverse of a linear, quadratic, exponential, or logarithmic function and analyze its graph	• determine restrictions on the domain of a function in order for its inverse to be a function, given the graph or equation
SO 2–SO 5, SO 9, SO 12– SO 14	• determine an unknown parameter in a function, given information relating to one point on the graph of the function	
SO 7	• determine, without technology, the exact values of simple logarithmic expressions • estimate the value of a logarithmic expression using benchmarks • convert between $y = b^x$ and $\log_b y = x$	• convert between exponential and logarithmic forms involving more than two steps
SO 8	• simplify and/or expand logarithmic expressions using the laws of logarithms	

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

SO 9	• sketch and analyze (domain, range, intercepts, asymptote) the graphs of exponential or logarithmic functions and their transformations	
SO 10	• solve exponential equations that: – can be simplified to a common base – cannot be simplified to a common base and whose exponents are monomials	• solve exponential equations that cannot be simplified to a common base, where the exponents are not monomials, or where there is a numerical coefficient
	• solve logarithmic equations but are not required to recognize when a solution is extraneous	• solve logarithmic equations and recognize when a solution is extraneous
	• solve exponential and logarithmic real-world application problems	
	• solve for a value, such as an earthquake intensity, in comparison problems	• solve for an exponent in comparison problems
SO 11	• identify whether a binomial is a factor of a given polynomial • completely factor a polynomial of degree 3, 4, or 5	
SO 12	• identify and explain whether a given function is a polynomial function • find the zeros of a polynomial function and explain their relationship to the *x*-intercepts of the graph and the roots of an equation • sketch and analyze polynomial functions (in terms of multiplicities, *y*-intercept, domain and range, etc.)	
	• provide a partial solution to solve a problem by modelling a given situation with a polynomial function	• provide a complete solution to a problem by modelling a given situation with a polynomial function
	• determine the equation of a polynomial function in factored form, given its graph and/or key characteristics	

SO 13	• sketch and analyze (in terms of domain, range, invariant points, *x*- and *y*-intercepts) $y=\sqrt{f(x)}$, given the graph or equation of $y=f(x)$ • find the zeros of a radical function graphically and explain their relationship to the *x*-intercepts of the graph and the roots of an equation • determine the equation of a radical function given its graph and/or key characteristics	• determine the equation of a radical function involving all three types of transformations: reflection, stretch, and translation, given its graph and/or key characteristics
SO 14	• sketch and analyze rational functions (in terms of vertical asymptotes, horizontal asymptote, *x*-coordinate of a point of discontinuity, domain, range, *x*- and *y*-intercepts) • find the zeros of a rational function graphically and explain their relationship to the *x*-intercepts of the graph and the roots of an equation • determine the equation of a rational function given its graph and/or key characteristics	• determine the *y*-coordinate of a point of discontinuity of a rational function • determine the equation of a rational function containing a point of discontinuity, given its graph and/or key characteristics
SO 1–SO 14	• participate in and contribute toward the problem-solving process for problems involving relations and functions studied in Mathematics 30-1	• complete the solution to problems involving relations and functions studied in Mathematics 30-1

Examples

Students who achieve the acceptable standard should be able to answer all of the following questions, except for any part of a question labelled ***SE***. Parts labelled ***SE*** are appropriate examples for students who achieve the standard of excellence.

Note: *In the multiple-choice questions that follow, * indicates the correct answer. Please be aware that the worked solutions show possible strategies; there may be other strategies that could be used.*

SO 1

1. Given the functions $f(x) = 7 - x$ and $g(x) = 2x + 1$, sketch the graph of $h(x)$ for each question below **and** state the domain and range.

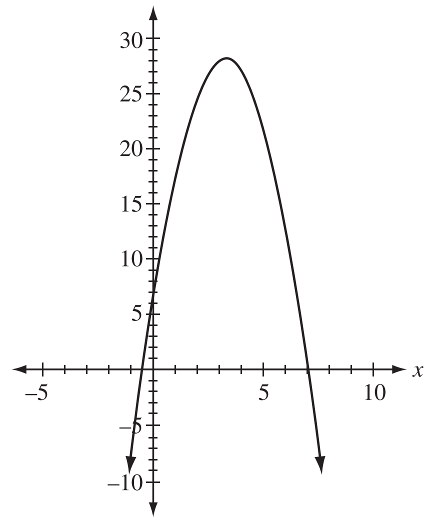
a) $h(x) = f(x)\,g(x)$

Possible solution:

$h(x) = (7 - x)(2x + 1)$

$h(x) = -2x^2 + 13x + 7$

Domain: $\{x \mid x \in R\}$ and Range: $\{y \mid y \le 28.125,\ y \in R\}$



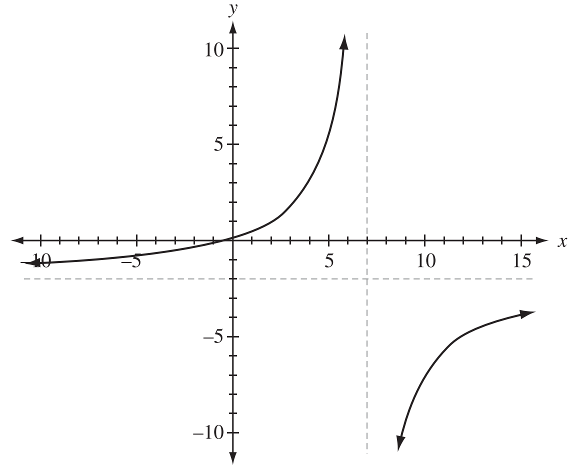
b) $h(x) = \left(\dfrac{g}{f}\right)(x)$

Possible solution:

$h(x) = \dfrac{2x + 1}{7 - x}$

Domain: $\{x \mid x \neq 7,\ x \in R\}$

and Range: $\{y \mid y \neq -2,\ y \in R\}$



To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

c) $h(x) = g(f(x))$

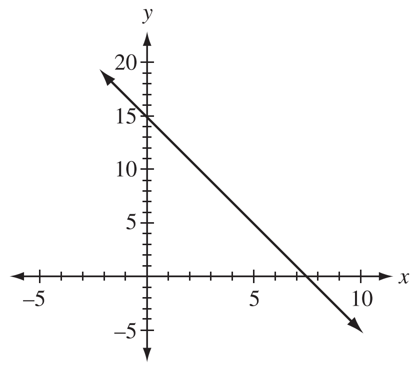
Possible solution:

$h(x) = g(7 - x)$

$h(x) = 2(7 - x) + 1$

$h(x) = 15 - 2x$

Domain: $\{x \mid x \in R\}$ and Range: $\{y \mid y \in R\}$



Use the following information to answer numerical-response question 2.

For functions $f(x) = \sqrt{x - 1}$, $g(x) = x^2 + 3$, $h(x) = 2x - 5$, and $k(x)$, where $k(x) = (h \circ g \circ f)(x)$, the simplified equation for $k(x)$ can be written in the form $k(x) = ax - b$, where $x \geq 1$.

Numerical Response

SE

SO 1

2. The value of ***a*** is ___________. (Record in the **first** box)
The value of ***b*** is ___________. (Record in the **second** box)

Solution: 21

$k(x) = h\left(g\left(\sqrt{x - 1}\right)\right)$

$k(x) = h\left(\left(\sqrt{x - 1}\right)^2 + 3\right)$

$k(x) = h(x + 2)$

$k(x) = 2(x + 2) - 5$

$k(x) = 2x - 1$

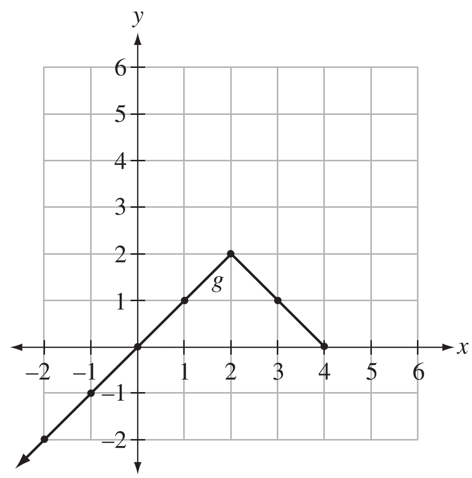
Note: This item is SE since it involves two compositions.

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

Use the following information to to answer numerical-response question 3.

The table shown below describes the function $y = f(x)$ and the graph shows the function $y = g(x)$.

x	***f(x)***
–1	undefined
0	4
1	5
2	6
3	6
4	7
5	8



Numerical Response

SO 1

3. If $h(x) = \dfrac{f(x)}{g(x)}$, then the value of $h(3)$ is ___________

Solution: 6

$$h(x) = \frac{f(x)}{g(x)}$$

$$h(3) = \frac{f(3)}{g(3)}$$

$$= \frac{6}{1}$$

$$= 6$$

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

SO 1
SO 7

4. Given $f(x) = 7\log_2 x$ and $g(x) = |5 - 6x|$, determine the value of $f\big((f + g)(8)\big)$.

Possible solution: $f(8) + g(8) = 7\log_2(8) + |5 - 6(8)|$

$f(8) + g(8) = 21 + 43$

$f(8) + g(8) = 64$

$f\big((f + g)(8)\big) = f(64)$

$f(64) = 7\log_2(64)$

$f(64) = 42$

$\therefore f\big((f + g)(8)\big) = 42$

Use the following information to answer numerical-response question 5.

Alex is given the following list of functions, where $b > 1$. She is asked to determine a new function, $h(x)$, which is the quotient of two different functions below and where the domain of $h(x)$ is $\{x \mid x \in R\}$.

Function 1	$y = x - b$
Function 2	$y = x^2 + b$
Function 3	$y = x^3 + b$
Function 4	$y = \log_b x$
Function 5	$y = b^x$
Function 6	$y = \sqrt{x + b}$

Numerical Response

SO 1
SO 9
SO 12
SO 13
SO 14

5. If $h(x) = \dfrac{f(x)}{g(x)}$ and Alex selects Function 1 for $f(x)$, then the two functions that she could select for $g(x)$ are numbered _____ and _____.

Solution: 25 or 52

Function 1 is the numerator and cannot also be the denominator as $f(x)$ and $g(x)$ must be different functions.

In order for $h(x)$ to have the domain $\{x \mid x \in R\}$, $g(x)$ cannot equal zero with $b > 1$, and $g(x)$ must have a domain of $\{x \mid x \in R\}$. Only functions 2 and 5 fit this description.

Function 3 becomes zero when $x = \sqrt[3]{-b}$.

Function 4 has a limited domain of $x > 0$.

Function 6 becomes zero when $x = -b$ and has a limited domain of $x \geq -b$.

SE

SO 1
SO 14

6. Given $f(x) = x^2 - 7x$, $g(x) = x - 2$, and $h(x) = \frac{2x^2 + x}{x - 2}$, determine a simplified equation for $j(x)$, given that $j(x) = \frac{f(x)}{g(x)} + h(x)$.

Possible solution: $j(x) = \frac{x^2 - 7x}{x - 2} + \frac{2x^2 + x}{x - 2}$

$$j(x) = \frac{3x^2 - 6x}{x - 2}$$

$$j(x) = \frac{3x(x - 2)}{x - 2}$$

$$j(x) = 3x, \quad x \neq 2$$

Note: This item is SE because it involves two operations with functions.

Use the following information to answer question 7.

> Two functions are shown below.
>
> $f(x):\{(1,4), (2,6), (3,5), (4,2)\}$
>
> $g(x):\{(1,5), (3,2), (5,8), (6,3)\}$

SO 1

7. The value of $(g \circ f)(3)$ is

A. 6
* **B.** 8
C. 9
D. 10

SE

SO 1

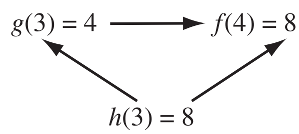
8. Given that Point A (3, 4) lies on the graph of $g(x)$, and Point A' (3, 8) lies on the graph of $h(x)$, where $h(x) = (f \circ g)(x)$, the corresponding point that lies on the graph of $f(x)$ must be _____ .

Possible solution: The x-coordinate of $f(x)$ is the y-coordinate of $g(x)$, 4.

The y-coordinate of $f(x)$ is the y-coordinate of $h(x)$, 8.

$\therefore (4, 8)$ is the corresponding point on $f(x)$.

Possible solution:



$\therefore (4, 8)$ is the corresponding point on $f(x)$.

Note: This item is SE since it requires the solution to problems involving relations and functions studied in Mathematics 30–1 as indicated in the last bullet on page 13.

Use the following information to answer numerical-response question 9.

Given that $f(x) = \sqrt{x-3}$ and $g(x) = \dfrac{x^2}{x^2 - 25}$, students are asked to select the domain of the graph of $y = h(x)$, where $h(x) = g\big(f(x)\big)$, from the list provided in the table below.

Reference Number	Domain
2	$\{x \mid x \geq 3, x \in R\}$
3	$\{x \mid x \neq 28, x \in R\}$
4	$\{x \mid x \geq 3, x \neq 28, x \in R\}$
5	$\{x \mid x \geq 3, x \neq 5, x \in R\}$
6	$\{x \mid x \geq 3, x \neq 5, x \neq 28, x \in R\}$

Numerical Response

SE

SO 1

9. The reference number that corresponds to the domain of the graph of $h(x) = g\big(f(x)\big)$ is ______ .

Solution: 4

$$g\big(f(x)\big) = \frac{\left(\sqrt{x-3}\right)^2}{\left(\sqrt{x-3}\right)^2 - 25}$$

For the domain, the radicand must be greater than or equal to zero

$$x - 3 \geq 0$$

and the denominator must not equal zero.

$$\left(\sqrt{x-3}\right)^2 - 25 \neq 0$$

$$g\big(f(x)\big) = \frac{x-3}{x-28}$$

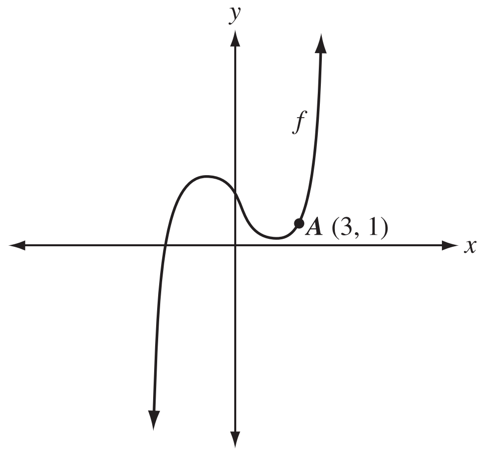
The domain of $y = f(x)$ is $x \geq 3$, and $y = h(x)$ is undefined at $x = 28$.

$\therefore \{x \mid x \geq 3, x \neq 28, x \in R\}$ and the correct reference number is 4.

Note: This item is SE since it requires the domain of a function that is the composition of two functions.

Use the following information to answer question 10.

The graph of function f is shown below. Function f is transformed to create function g.



SO 2

10. If $g(x) - 2 = f(x + 5)$, then which quadrant contains the corresponding point, A′, on the graph of function g?

A. Quadrant I
***B.** Quadrant II
C. Quadrant III
D. Quadrant IV

Numerical Response

SO 2

11. *The function* $f(x) = |x - 2| + 3$ *is transformed into the function* $g(x) = |x + 2| + 1$. *The transformations that will transform* $y = f(x)$ *into* $y = g(x)$ *are a translation of* _____ *units* _____ *and a translation of* _____ *units* _____.

Use the following code to complete the sentence above.

Reference Number	Numerical Value
1	1
2	2
3	3
4	4

Reference Number	Translation Direction
5	left
6	right
7	up
8	down

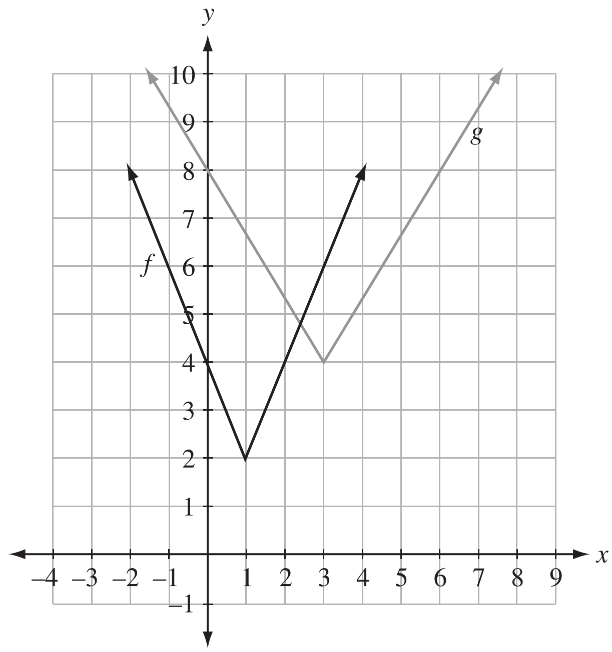
Possible solution: 4528 or 2845

SO 2 **12.** What transformations of the function $f(x) = x^3$ are described by the mapping notation $(x, y) \rightarrow (x - 4, y + 9)$?

A. 4 units right and 9 units down
B. 4 units left and 9 units down
C. 4 units right and 9 units up
***D.** 4 units left and 9 units up

Use the following information to answer question 13.

The graph of the function $y = f(x)$ is transformed to produce the graph of the function $y = g(x)$.



SO 3 **13.** An equation for $g(x)$ in terms of $f(x)$ is

A. $g(x) = \frac{1}{2}f(3x)$

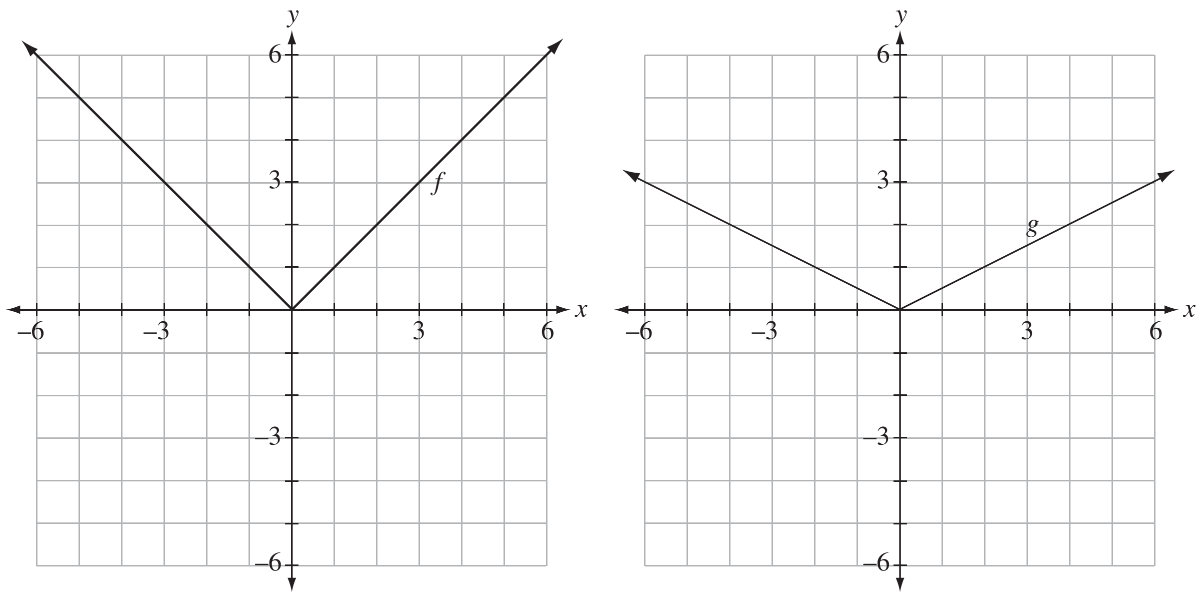
B. $g(x) = 2f(3x)$

C. $g(x) = \frac{1}{2}f\left(\frac{1}{3}x\right)$

***D.** $g(x) = 2f\left(\frac{1}{3}x\right)$

Use the following information to answer question 14.

The graphs of $f(x) = |x|$ and $y = g(x)$ are shown below. The graph of $f(x)$ undergoes a single transformation to become the graph of $g(x)$.



SO 3 **14.** Determine two possible equations for the function $g(x)$.

Possible solution: $g(x) = \frac{1}{2}|x|$ and $g(x) = \left|\frac{1}{2}x\right|$

The single transformation can be either a vertical stretch by a factor of $\frac{1}{2}$ about the x-axis or a horizontal stretch by a factor of 2 about the y-axis.

Use the following information to answer numerical-response question 15.

The ordered pairs below represent possible transformations of Point $P(a, b)$ on the graph of the function $y = f(x)$.

Point 1: $(4a, b)$	**Point 3:** $(a, -b)$	**Point 5:** $\left(\frac{a}{4}, b\right)$
Point 2: $(-a, b)$	**Point 4:** $\left(a, \frac{b}{4}\right)$	**Point 6:** $(a, 4b)$

Numerical Response

SO 3
SO 5

15. If $y = f(x)$ undergoes the following single transformations, identify the coordinates of the corresponding Point P′ on the new graph.

The corresponding point on the function $y = -f(x)$ is point number __________.

The corresponding point on the function $y = f\left(\frac{1}{4}x\right)$ is point number __________.

The corresponding point on the function $y = \frac{1}{4}f(x)$ is point number __________.

The corresponding point on the function $y = f(-x)$ is point number __________.

Solution: 3142

Use the following information to answer question 16.

The graph of $y = f(x)$ is reflected in the x-axis, stretched vertically about the x-axis by a factor of $\frac{1}{3}$, and stretched horizontally about the y-axis by a factor of 4 to create the graph of $y = g(x)$.

SO 3
SO 5

16. For Point $A(-3, 6)$ on the graph of $y = f(x)$, the corresponding image point, A', on the graph of $y = g(x)$ is

A. (9, 24)
B. (–12, –18)
C. (1, 24)
***D.** (–12, –2)

Use the following information to answer numerical-response question 17.

The graph of $y = f(x)$ is transformed into the graph of $g(x) + 4 = 2f(x - 3)$. The domain and range of each function are shown below.

	Domain	Range
Graph of $f(x)$	[–1, 3]	[2, 6]
Graph of $g(x)$	$[a, b]$	$[c, d]$

Numerical Response

SO 4

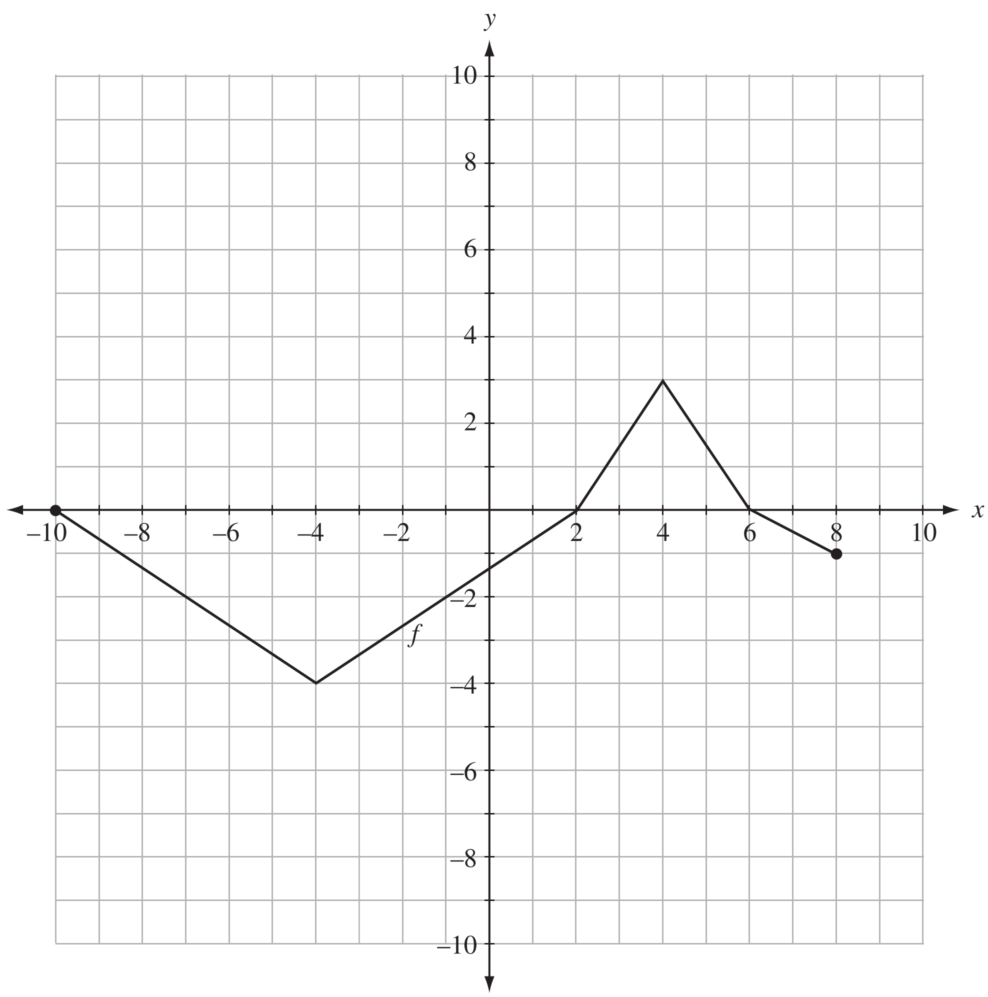
17. For the graph of $g(x)$, the values of $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, and $\boldsymbol{d}$ are, respectively, _____, _____, _____, and _____.

Solution: 2608

Since $(x, y) \rightarrow (x + 3, 2y - 4)$, Domain: [2, 6] and Range: [0, 8] for the graph of $g(x)$.

Use the following information to answer question 18.

The graph of $y = f(x)$ below is reflected in the y-axis, vertically stretched by a factor of 2 about the x-axis, horizontally stretched by a factor of $\frac{1}{2}$ about the line $x = 0$, and then translated 3 units up.

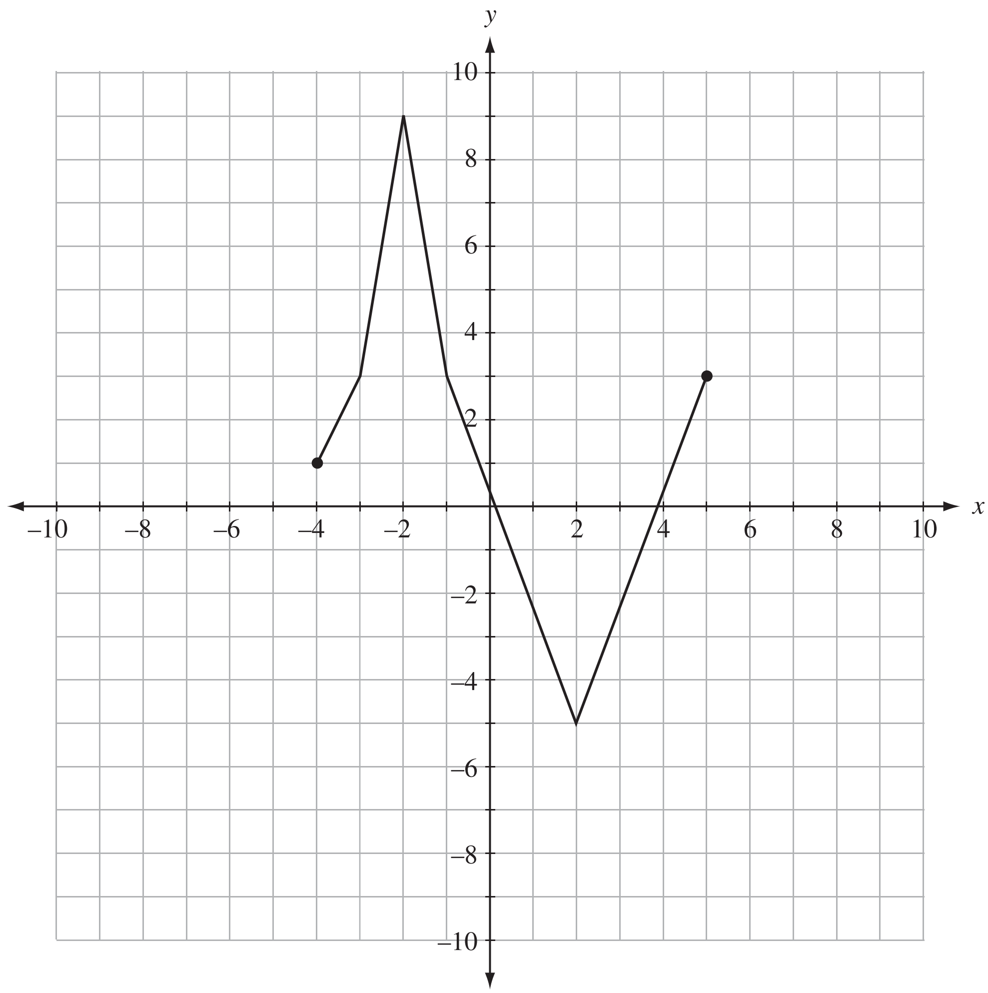


SE

SO 4
SO 5

18. Sketch the graph of the new function.

Possible solution:



$(x, y) \to \left(-\frac{1}{2}x,\ 2y + 3\right)$

Note: This item is SE since it involves a reflection, a stretch, and a translation.

SO 5

19. When the graph of $y = -x^2 + 4$ is reflected in the y-axis, the equation of the transformed function will be

A. $y = x^2 + 4$

***B.** $y = -x^2 + 4$

C. $y = x^2 - 4$

D. $y = -x^2 - 4$

Use the following information to answer numerical-response question 20.

The graph of $y = \sqrt{x}$ is transformed into the graph of $y = \sqrt{-\frac{1}{2}x - 4}$.

A list of possible transformations is shown below.

Reference Number	Transformation
1	Horizontal stretch by a factor of $\frac{1}{2}$ about the *y*-axis
2	Horizontal stretch by a factor of 2 about the *y*-axis
3	Reflection in the *x*-axis
4	Reflection in the *y*-axis
5	Horizontal translation of 4 left
6	Horizontal translation of 4 right
7	Horizontal translation of 8 left
8	Horizontal translation of 8 right

Numerical Response

SE

SO 4
SO 5
SO 13

20. A sequence of transformations from the list above that describe the changes to the graph of $y = \sqrt{x}$ is _____, _____, and _____.

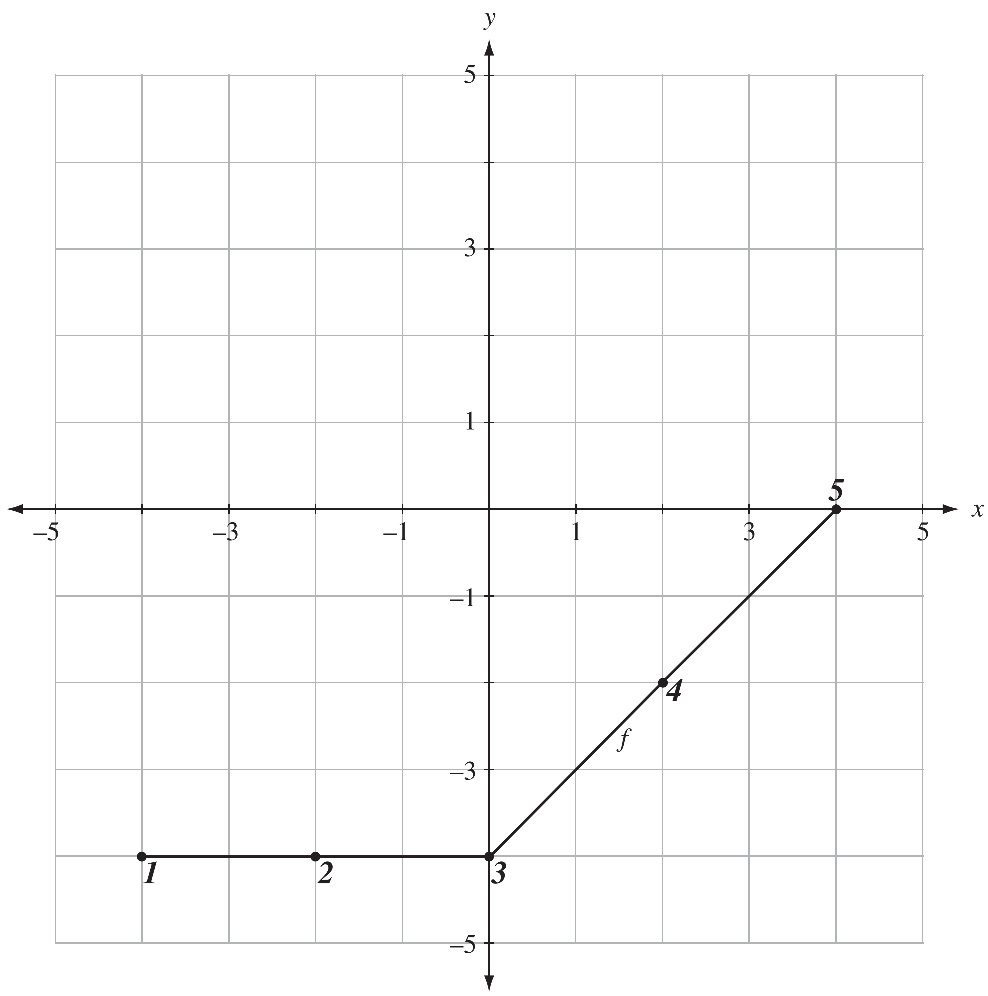
Possible solution: 247 or 427

The graph of $y = \sqrt{x}$ is reflected in the *y*-axis, horizontally stretched by a factor of 2 about the *y*-axis, and then translated 8 units left.

Note: This item is SE since the transformation involves factoring the *b* value and since it contains a reflection, a stretch, and a translation.

Use the following information to answer question 21.

The graph of $y = f(x)$ is shown below.



SO 5
SO 6

21. For each transformation of $y = f(x)$ indicated below, the invariant point exists at point number:

______	______	______
$y = -f(x)$	$y = f(-x)$	$x = f(y)$

Solution: 531

SO 5
SO 9

22. The graph of $y = 3^{x+2}$ is reflected in the line $y = x$. The equation of the new graph is

- ***A.** $y = \log_3 x - 2$
- **B.** $y = \log_3 x + 2$
- **C.** $y = \log_3 (x - 2)$
- **D.** $y = \log_3 (x + 2)$

Use the following information to answer numerical-response question 23.

> The inverse of $f(x) = 2x - 3$ can be written as $f^{-1}(x) = \frac{a}{2}x + \frac{b}{2}$.

Numerical Response

SO 6

23. The values of ***a*** and ***b*** are, respectively, _____ and _____.

Solution: 13

$$x = 2y - 3$$
$$x + 3 = 2y$$
$$\frac{x+3}{2} = y$$
$$\therefore f^{-1}(x) = \frac{1}{2}x + \frac{3}{2}$$

SE

SO 6

24. A restriction on the domain of $f(x) = x^2 + 4$, such that its inverse is also a function, could be

- **A.** $\{x \mid x \geq -4, x \in R\}$
- ***B.** $\{x \mid x \geq 0, x \in R\}$
- **C.** $\{x \mid x \leq 2, x \in R\}$
- **D.** $\{x \mid x \leq 4, x \in R\}$

Note: This item is SE since it involves a restriction on the domain in order for the inverse to be a function.

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

Numerical Response

SO 7

25. The value of $\log_5 625 + 3\log_7 49 + \log_2 \frac{1}{16} + \log_b b + \log_a 1$ is __________.

Solution: $4 + 3(2) + -4 + 1 + 0 = 7$

SE

SO 7

26. The equation $m\log_p n + 5 = q$ can be written in exponential form as

A. $p^{(q-5)} = mn$

***B.** $p^{(q-5)} = n^m$

C. $p^{(q-5)} = \frac{m}{n}$

D. $p^{(q-5)} = m^n$

Note: This item is SE since the conversion involves more than two steps.

SO 8

27. The expression $\left(3^{\log x}\right)\left(3^{\log x}\right)$ is equivalent to

***A.** $3^{\log x^2}$

B. $9^{\log x^2}$

C. $3^{(\log x)^2}$

D. $9^{(\log x)^2}$

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

SO 8 **28.** Written as a single logarithm, $2\log x - \frac{\log z}{2} + 3\log y$ is

***A.** $\log\left(\frac{x^2y^3}{\sqrt{z}}\right)$

B. $3\log\left(\frac{xy}{z}\right)$

C. $\log\left(\frac{x^2}{y^3\sqrt{z}}\right)$

D. $\log\left(x^2 - \sqrt{z} + y^3\right)$

SO 8 **29.** Given that $\log_3 a = 6$ and $\log_3 b = 5$, determine the value of $\log_3\left(9ab^2\right)$.

Possible solution: $\log_3 9 + \log_3 a + 2\log_3 b = 2 + 6 + 2(5) = 18$

Possible solution: $3^6 = a$ and $3^5 = b$

$\log_3\left(3^2 \times 3^6 \times \left(3^5\right)^2\right)$

$\log_3\left(3^2 \times 3^6 \times 3^{10}\right)$

$\log_3 3^{18}$

18

Use the following information to answer question 30.

A student's work to simplify a logarithmic expression is shown below, where $a > 1$.

Step 1 $2\log_a x^4 - 3\log_a x^2 + 4\log_a x^3$

Step 2 $\log_a x^8 - \log_a x^6 + \log_a x^{12}$

Step 3 $\log_a\left(\dfrac{x^8}{x^6 \times x^{12}}\right)$

Step 4 $\log_a\left(\dfrac{x^8}{x^{18}}\right)$

Step 5 $\log_a x^{10}$

SO 8 **30.** The student's **first** recorded error is in Step

A. 2
***B.** 3
C. 4
D. 5

SO 9 **31.** The equation of the asymptote for the graph of $y = \log_b(x - 3) + 2$, where $b > 1$, is

A. $y = 2$
B. $y = -2$
***C.** $x = 3$
D. $x = -3$

Use the following information to answer question 32.

A student sketched the graphs of $f(x) = \log_a(x + 3) - 7$ and $g(x) = a^{(x-2)} + 5$, where $a > 1$, on a coordinate plane. She also drew the asymptotes of the two graphs using dotted lines.

SO 9

32. The intersection point of the two dotted lines will be at

A. (3, 5)
***B.** (–3, 5)
C. (2, –7)
D. (–2, –7)

SE

SO 9

33. For the graph of $y = \log_b(3x + 12)$, where $b > 1$, the domain is

***A.** $\{x \mid x > -4,\ x \in R\}$
B. $\{x \mid x > 4,\ x \in R\}$
C. $\{x \mid x > -12,\ x \in R\}$
D. $\{x \mid x > 12,\ x \in R\}$

Note: This item is SE since the *b* value is not factored out of the binomial.

SO 9

34. The *y*-intercept on the graph of $f(x) = a^{(x+1)} + b$, where $a > 0$, $a \neq 1$, is

A. a
B. b
C. $1 + b$
***D.** $a + b$

SO 10 **35.** The solution to the equation $8^{(3x+4)} = 4^{(x-9)}$ is

A. $\frac{-13}{2}$

***B.** $\frac{-30}{7}$

C. $\frac{-17}{5}$

D. $\frac{-22}{7}$

Numerical Response

SE **SO 10** **36.** Solving the equation $3^{(2x+1)} = \left(\frac{1}{5}\right)^{(x-3)}$ algebraically, the solution, to the nearest hundredth, is _____.

Solution: 0.98

$$3^{(2x+1)} = 5^{(-x+3)}$$

$$(2x+1)\log 3 = (-x+3)\log 5$$

$$2x\log 3 + \log 3 = -x\log 5 + 3\log 5$$

$$2x\log 3 + x\log 5 = 3\log 5 - \log 3$$

$$x(2\log 3 + \log 5) = 3\log 5 - \log 3$$

$$x = \frac{3\log 5 - \log 3}{2\log 3 + \log 5}$$

$$x = 0.97978\ldots$$

$$x \approx 0.98$$

Note: This item is SE since the powers have no common base and because the exponents are not monomials.

SE

SO 10

37. Solve algebraically $\log_7(x+1)+\log_7(x-5)=1.$

Possible solution:

$$\log_7(x+1)+\log_7(x-5)=1$$
$$\log_7\left(x^2-4x-5\right)=1$$
$$7=x^2-4x-5$$
$$0=x^2-4x-12$$
$$0=(x-6)(x+2)$$
$$x=6 \text{ or } x=-2$$
$$x=-2 \text{ is an extraneous root}$$
$$\therefore x=6$$

Note: This item is SE if students are required to recognize that a solution is extraneous.

Use the following information to answer numerical-response question 38.

Earthquake intensity is given by $I=I_0\times 10^M$, where I_0 is the reference intensity and M is the magnitude. An earthquake measuring 5.3 on the Richter scale is 125 times more intense than a second earthquake.

Numerical Response

SE

SO 10

38. The Richter scale measure of the second earthquake, to the nearest tenth, is _____.

Solution: 3.2

$$\frac{I_2}{I_1}=\frac{I_0\times 10^{5.3}}{I_0\times 10^x}$$
$$125=10^{5.3-x}$$
$$5.3-x=\log_{10}125$$
$$x=5.3-\log_{10}125$$
$$x\approx 3.2$$

Note: This item is SE since it requires solving for an exponent in a comparison problem.

SE

SO 10

39. The population of a particular town on January 1, 2008, was 15 000. If the population of this town on January 1, 2016, was 32 450, what is the average annual rate of increase?

A. 0.1%
B. 1.1%
***C.** 10%
D. 110%

SO 10

40. Jordan needs $6 000 to take his family on a trip. He is able to make an investment which offers an interest rate of 8%/a compounded semi-annually. How much should Jordan invest now, to the nearest dollar, so that he has enough money to go on a family trip in 3 years?

Possible solution: $y = ab^{\frac{t}{p}}$

$$6\,000 = a(1.04)^{\frac{3}{1/2}}$$

$$a = 4\,741.887\,15...$$

Jordan should invest $4 742 now.

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

SO 11 **41.** Express the following polynomials in factored form.

a) $P(x) = x^3 - x^2 - 8x + 12$

Possible solution: $P(2) = 2^3 - 2^2 - 8(2) + 12 = 0 \rightarrow (x - 2)$ is a factor

–2	1	–1	–8	12
	↓	–2	–2	12
	1	1	–6	0

$x^2 + x - 6 = (x + 3)(x - 2)$

$\therefore\ P(x) = (x - 2)^2(x + 3)$

b) $P(x) = 2x^4 + 3x^3 - 17x^2 - 27x - 9$

Possible solution: $P(-1) = 2(-1)^4 + 3(-1)^3 - 17(-1)^2 - 27(-1) - 9 = 0 \rightarrow (x + 1)$ is a factor

1	2	3	–17	–27	–9
	↓	2	1	–18	–9
	2	1	–18	–9	0

$Q(x) = 2x^3 + x^2 - 18x - 9$

$Q(3) = 2(3)^3 + 3^2 - 18(3) - 9 = 0 \rightarrow (x - 3)$ is a factor

–3	2	1	–18	–9
	↓	–6	–21	–9
	2	7	3	0

$2x^2 + 7x + 3 = (2x + 1)(x + 3)$

$\therefore\ P(x) = (x + 1)(x - 3)(2x + 1)(x + 3)$

SO 11 **42.** The polynomial function $P(x) = 4x^4 - x^3 - 8x^2 - 40$ has a linear factor of $x + 2$.

The remaining cubic factor is

A. $4x^3 + 7x^2 + 6x - 20$
B. $4x^3 + 7x^2 + 6x + 12$
***C.** $4x^3 - 9x^2 + 10x - 20$
D. $4x^3 - 9x^2 + 10x - 60$

Use the following information to answer numerical-response question 43.

The binomial $x + 2$ is a factor of $f(x) = x^3 + 3x^2 + kx + 4$.

Numerical Response

SO 11 **43.** The value of $\boldsymbol{k}$ is _____.

Solution: $f(-2) = (-2)^3 + 3(-2)^2 + k(-2) + 4$

$0 = -2k + 8$

$k = 4$

SO 11
SO 12 **44.** *If $P(x)$ is a polynomial function where $P\left(-\frac{2}{3}\right) = 0$ and $P(0) = 12$, then* ___***i***___ *is a factor of $P(x)$, and* ___***ii***___ *is a constant term in the equation of $P(x)$.*

The statement above is completed by the information in row

Row	*i*	*ii*
A.	$(3x - 2)$	12
***B.**	$(3x + 2)$	12
C.	$(3x - 2)$	–12
D.	$(3x + 2)$	–12

Use the following information to answer numerical-response question 45.

For $P(x) = 2x^3 - 3x^2 - 10x + 3$, the zeros can be written as $x = m$ and $x = \dfrac{n \pm \sqrt{p}}{4}$, where $m, n, p \in I$.

Numerical Response

SO 11 **45.** The value of ***p*** is ______ .

Solution: 17

If $x = 3$, then

$$P(3) = 2(3)^3 - 3(3)^2 - 10(3) + 3$$
$$= 0$$

Therefore, $(x - 3)$ is a factor.

–3	2	–3	–10	3
	↓	–6	–9	3
	2	3	–1	0

Since the remaining factor $(2x^2 + 3x - 1)$ cannot be factored, use the quadratic formula to determine the remaining roots.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

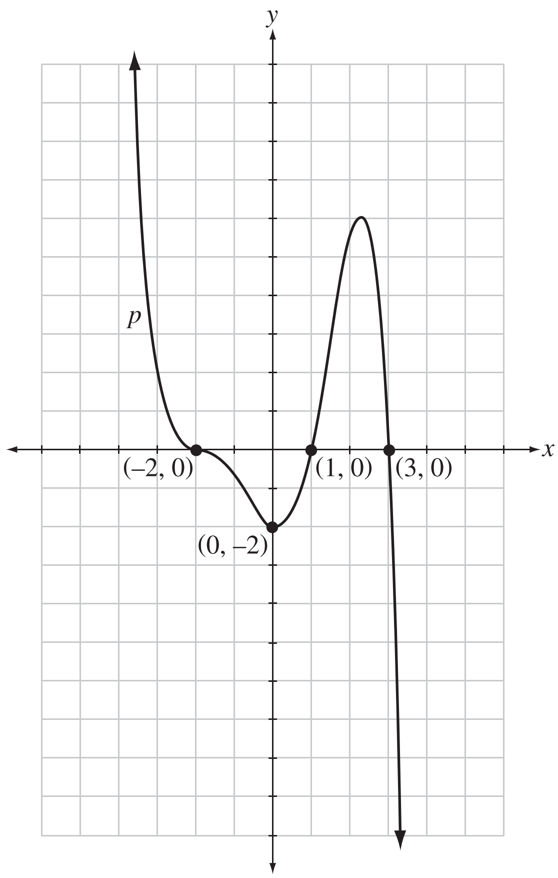
$$x = \frac{-3 \pm \sqrt{3^2 - 4(2)(-1)}}{2(2)}$$

$$x = \frac{-3 \pm \sqrt{17}}{4}$$

The value of ***p*** is 17.

SO 12 **46.** Sketch the graph of a fifth-degree polynomial function with one real zero of multiplicity 3 and with a negative leading coefficient.

Possible solution:



To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

SO 12 **47.** Given the function $y = \frac{1}{4}(x-2)(2x+5)(x+4)^2$:

a) Accurately sketch the graph, and label any key points.

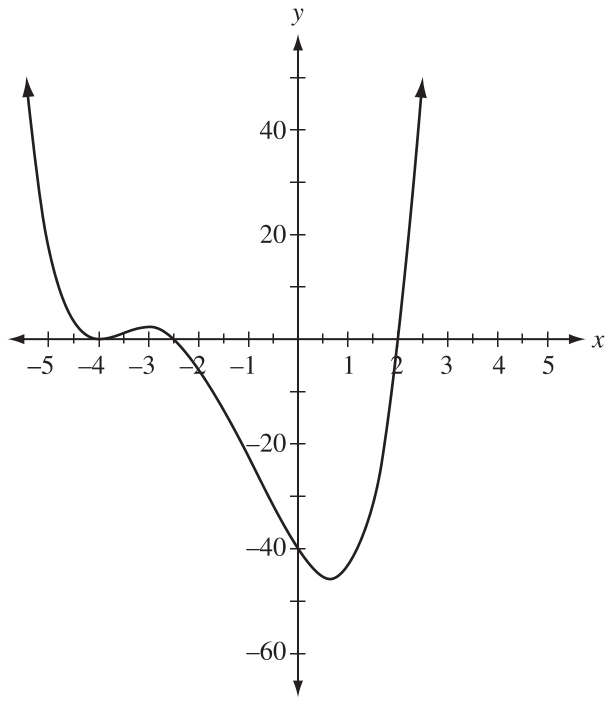
Possible solution:

Key points:

x-intercepts at 2, $-\frac{5}{2}$, –4

y-intercept at –40

Using technology, the minimum value is at approximately –45.976 when $x = 0.659$.



b) State the domain and range.

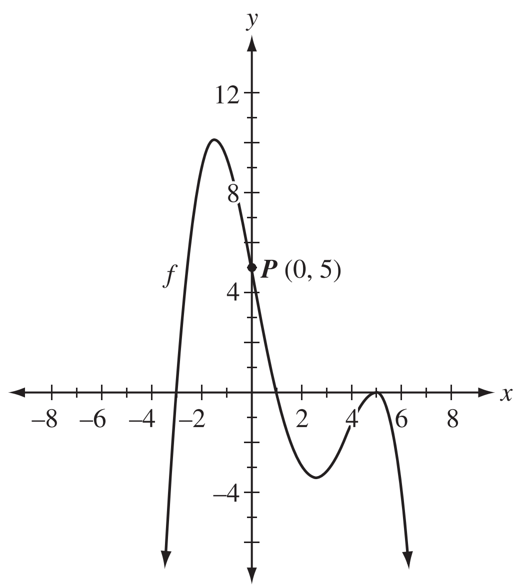
Possible solution: Domain: $\{x \mid x \in R\}$ and Range: $\{y \mid y \geq -45.976, y \in R\}$

c) Determine the zeros of the function.

Possible solution: zeros at $x = 2$, $x = -\frac{5}{2}$, $x = -4$ with multiplicity 2

Use the following information to answer numerical-response question 48.

The graph of the polynomial function $y = f(x)$ is shown below. The equation of this graph can be written in the form $y = -\frac{1}{a}(x-1)(x+b)(x-c)^2$.



Numerical Response

SO 12 **48.** The value of ***a*** is __________.

Solution: 15

Since the zeros of the function are –3, 1, and 5 (multiplicity 2), the equation that represents the function is $f(x) = -\frac{1}{a}(x+3)(x-1)(x-5)^2$. Use the y-intercept (0, 5) to find the leading coefficient.

$$5 = -\frac{1}{a}(0+3)(0-1)(0-5)^2$$

$$5 = -\frac{1}{a}(-75)$$

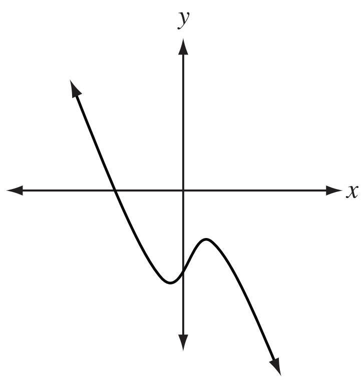
$$-\frac{1}{15} = -\frac{1}{a}$$

$$\therefore f(x) = -\frac{1}{15}(x+3)(x-1)(x-5)^2$$

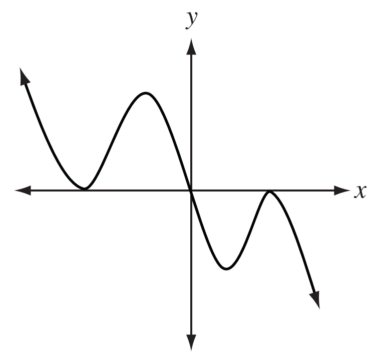
Use the following information to answer numerical-response question 49.

The graphs of four polynomial functions are shown below.

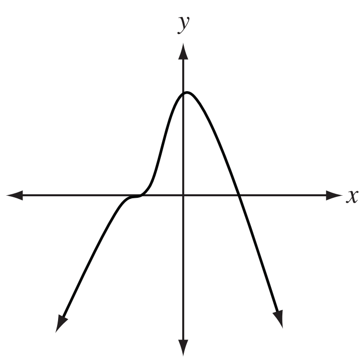
Graph 1



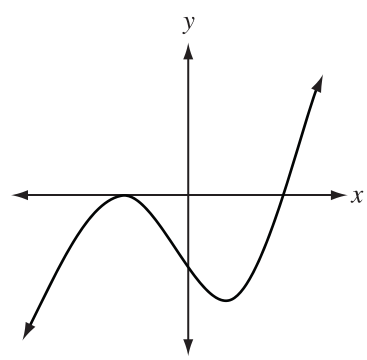
Graph 2



Graph 3



Graph 4



Numerical Response

SO 12 **49.** Match three of the graphs numbered above with a statement below that best describes the graph's function.

The graph whose function has a positive leading coefficient is graph number ________.

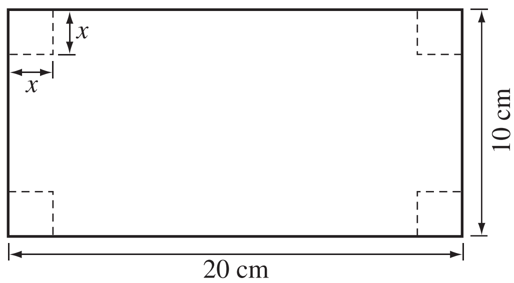
The graph of a function that has two different zeros, each with multiplicity 2, is graph number ________.

The graph that could be of a degree 4 function is graph number ________.

Solution: 423

Use the following information to answer question 50.

A box with no lid is made by cutting four squares of side length x from each corner of a 10 cm by 20 cm rectangular sheet of metal.



SE

SO 12

50. Using the information above, follow the directions below.

a) Determine an expression that represents the volume of the box.

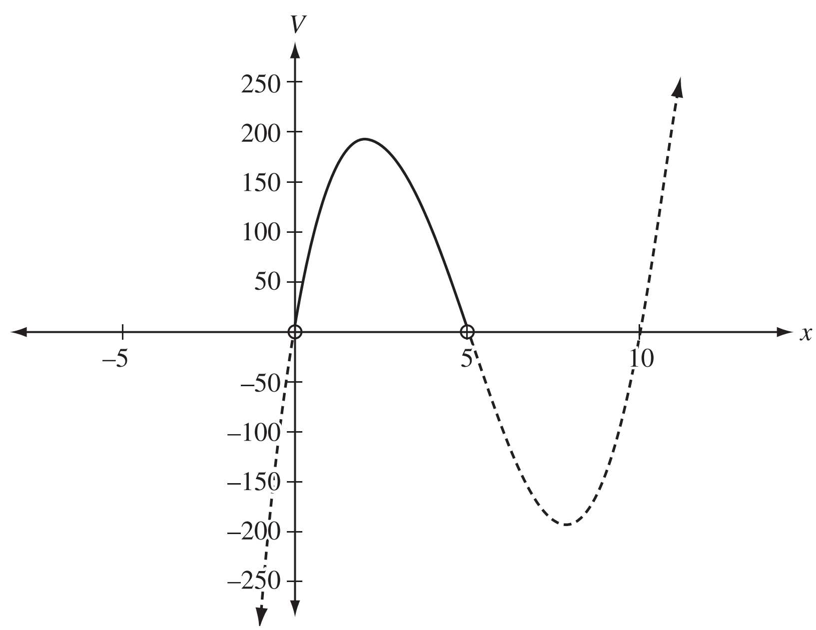
Possible solution: $V = x(10 - 2x)(20 - 2x)$

b) Sketch the graph of the function and state the restriction on the domain.

Possible solution:

Where $0 < x < 5$

Since the volume must be positive and one dimension of the sheet is 10 cm, the value of x cannot exceed 5 cm.



c) Determine the value of x, to the nearest hundredth of a centimetre, that gives the maximum volume.

Possible solution: Using technology, the volume is a maximum when $x \approx 2.11$ cm.

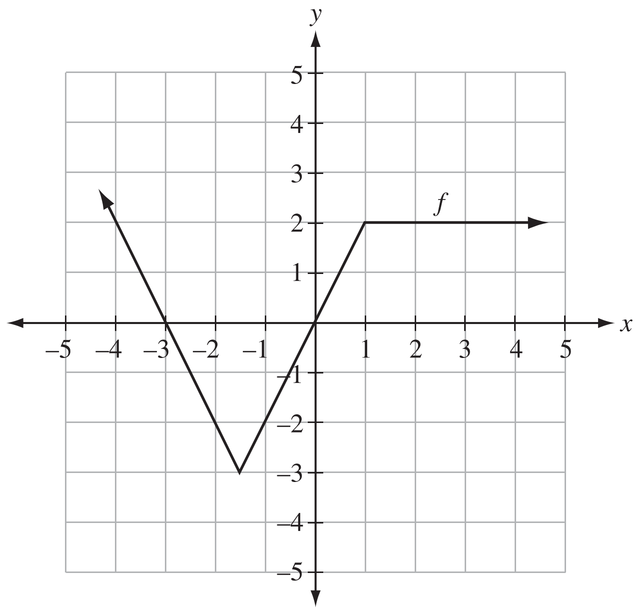
d) What is the maximum volume of the box, to the nearest cubic centimetre?

Possible solution: Using technology, the maximum volume of the box is 192 cm^3.

Note: This item is SE since it requires a complete solution to a problem by modelling a given situation with a polynomial function.

Use the following information to answer question 51.

The graph of the function $y = f(x)$ is shown below.

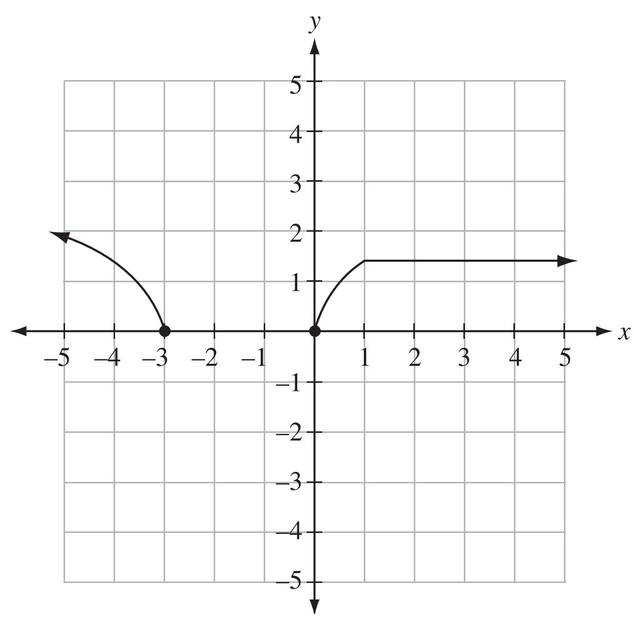


SO 13 **51.** Sketch the graph of $y = \sqrt{f(x)}$ and state the domain and range.

Possible solution:

Domain: $(-\infty, -3]$ or $[0, \infty)$

Range: $[0, \infty)$



Use the following information to answer numerical-response question 52.

The graph of the function $f(x) = \frac{1}{2}x - 3$ is transformed into $y = \sqrt{f(x)}$. The invariant points of the graph are located at (a, b) and (c, d).

Numerical Response

SO 13 **52.** The values of $\boldsymbol{a}$, $\boldsymbol{b}$, $\boldsymbol{c}$, and $\boldsymbol{d}$ are, respectively, _____, _____, _____, and _____.

Solution: 6081 or 8160

When $y = f(x)$ is transformed into $y = \sqrt{f(x)}$, the invariant points exist where $f(x) = 0$ and $f(x) = 1$.

$\therefore$ The coordinates of the invariant points are $(6, 0)$ and $(8, 1)$.

Numerical Response

SO 13 **53.** For the function $f(x) = -2\sqrt{x+4} + 3$, the x-intercept is equal to $-k$. The value of $\boldsymbol{k}$, to the nearest hundredth, is _____.

Solution: The value of k is 1.75.

SO 14 **54.** Sketch the graph of the following functions and state the following characteristics for each function below: domain, *x*- and *y*-intercepts, and the equations of the vertical asymptotes.

a) $y = \dfrac{3x}{x^2 + 2x - 8}$

Possible solution:

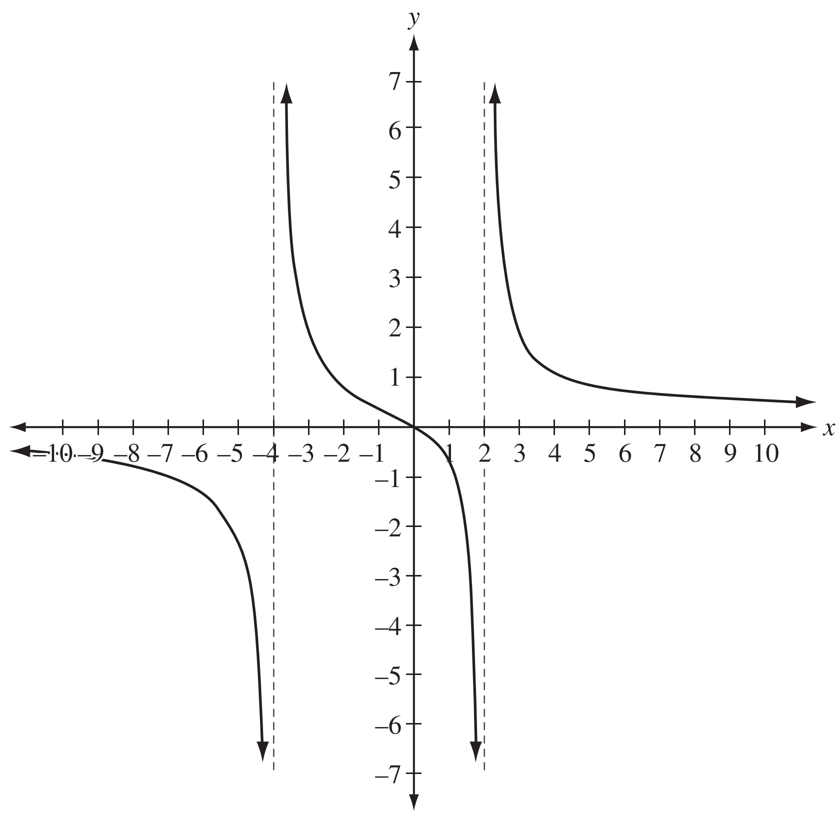
$y = \dfrac{3x}{(x+4)(x-2)}$

Domain: $\{x \mid x \neq -4, 2, x \in R\}$

x-intercept: 0

y-intercept: 0

Equations of vertical asymptotes:
$x = -4$ and $x = 2$



To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

b) $y = \dfrac{x+3}{x^2-9}$

Possible solution:

$$y = \frac{x+3}{(x+3)(x-3)}$$

$$y = \frac{1}{x-3}$$

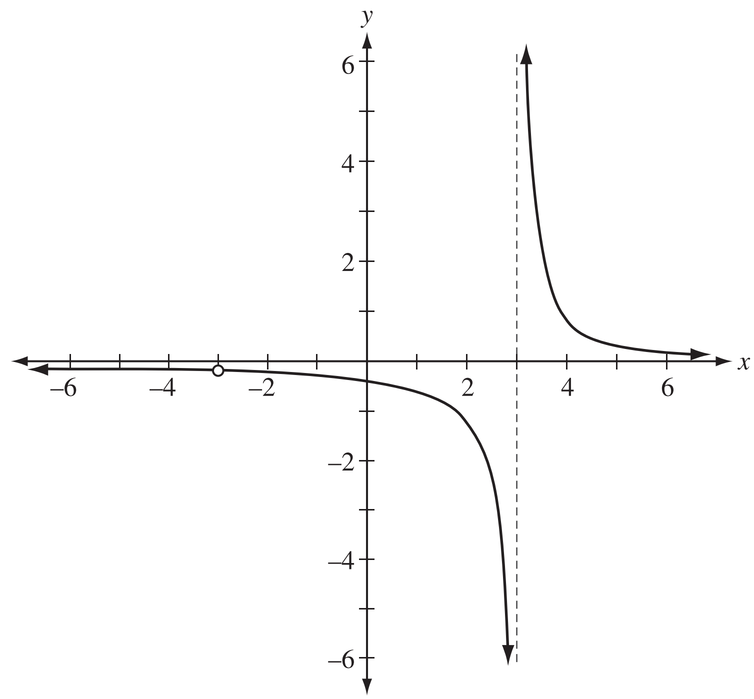
Domain: $\{x \mid x \neq \pm 3, x \in R\}$

No x-intercept

y-intercept: $-\dfrac{1}{3}$

Equation of vertical asymptote: $x = 3$

There is a point of discontinuity at $x = -3$.



Numerical Response

SO 2

SO 14

55. *If* $f(x) = \frac{x+6}{x+2}$, *then the equation of the vertical asymptote is numbered* _____ *and the equation of the horizontal asymptote is numbered* _____.

Use the following code to complete the sentence above.

Possible Equations

1 $x = -4$

2 $x = -2$

3 $x = 2$

4 $x = 4$

5 $y = -1$

6 $y = 0$

7 $y = 1$

Possible solution: 27

$$f(x) = \frac{x+6}{x+2}$$

$$f(x) = \frac{x+2+4}{x+2}$$

$$f(x) = \frac{x+2}{x+2} + \frac{4}{x+2}$$

$$f(x) = 1 + \frac{4}{x+2} \text{ or } f(x) = \frac{4}{x+2} + 1$$

When compared to the graph of $y = \frac{1}{x}$, the graph of $y = f(x)$ is vertically stretched by a factor of 4 about the x-axis and translated 2 units to the left and 1 unit up. Therefore, the vertical asymptote will be located at $x = -2$, and the horizontal asymptote will be located at $y = 1$.

SE

SO 14

56. Determine the coordinates of the point of discontinuity on the graph of $f(x) = \dfrac{2x^2 - 15x + 7}{x - 7}$.

Possible solution: $f(x) = \dfrac{(2x - 1)(x - 7)}{x - 7}$

There is a point of discontinuity when $x = 7$.

$\therefore f(x) = 2x - 1$, for $x \neq 7$

$y = 2(7) - 1$

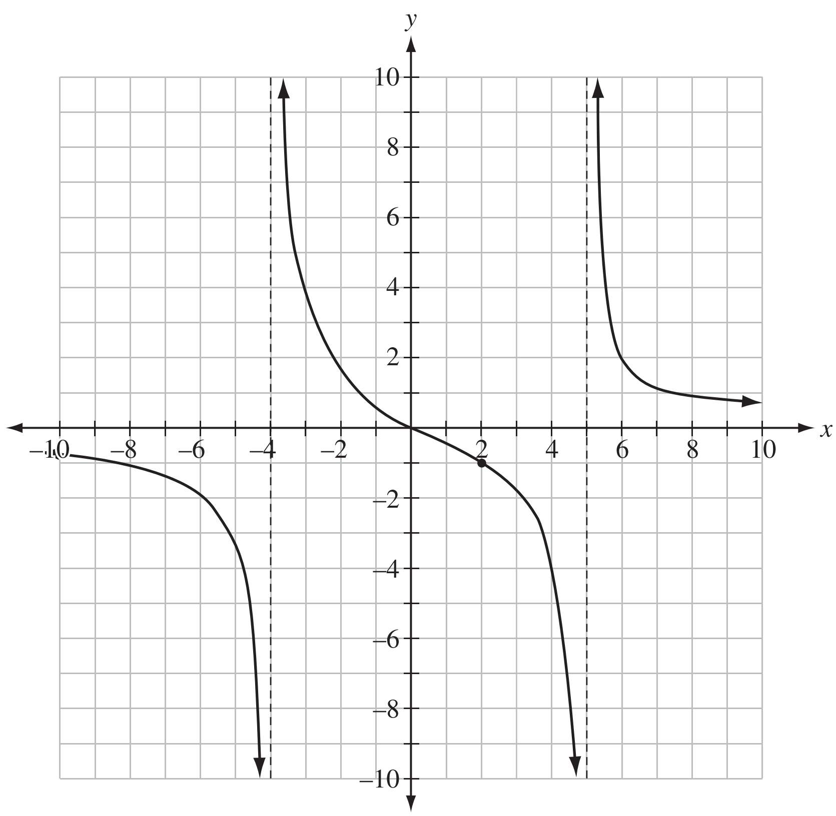
$y = 13$

The point of discontinuity is (7, 13).

Note: This item is SE since the y-coordinate of the point of discontinuity is required.

Use the following information to answer numerical-response question 57.

The graph of the function below can be expressed in the form $f(x) = \dfrac{ax}{x^2 - bx - c}$, where $f(2) = -1$.



Numerical Response

SO 14 **57.** The values of $\boldsymbol{a}$, $\boldsymbol{b}$, and $\boldsymbol{c}$ are _____, _____, and _____.

Possible solution: 9120

Since the vertical asymptotes exist at $x = -4$ and $x = 5$,

$$y = \frac{ax}{(x+4)(x-5)}.$$

Using the point (2, –1), $-1 = \dfrac{a(2)}{(2+4)(2-5)}$

$$18 = 2a$$

$$9 = a$$

$$\therefore y = \frac{9x}{x^2 - x - 20}$$

Trigonometry

General Outcome

Develop trigonometric reasoning.

General notes:

- Students need to differentiate between "exact value" and "approximate value" in problems.
- Students should be able to express exact value answers with either a rationalized or unrationalized denominator.
- Students should be able to perform, analyze, and describe transformations on sinusoidal functions.
- When appropriate, technology could include graphing calculators, computer graphing applications, and applets.
- The restricted domain may be something other than $0° \le \theta < 360°$ or $0 \le \theta < 2\pi$; for example, $-\pi \le \theta \le \frac{3\pi}{2}$.

Specific Outcomes

Specific Outcome 1

Demonstrate an understanding of angles in standard position, expressed in degrees and radians. **[CN, ME, R, V]**

Note:

- The understanding of radian measure $\theta = \frac{a}{r}$ is intended to be a part of this outcome.

(See examples 1-6 and 14.)

Specific Outcome 2

Develop and apply the equation of the unit circle. **[CN, R, V]**

Note:

- The intent of this outcome is that students have a deeper understanding of how points on the unit circle relate to the equation of the unit circle and the trigonometric ratios.

(See examples 6-9.)

Specific Outcome 3

Solve problems, using the six trigonometric ratios for angles expressed in radians and degrees.
[ME, PS, R, T, V] [ICT : C6–4.1]

(See examples 9-14 and 32.)

Specific Outcome 4

Graph and analyze the trigonometric functions sine, cosine, and tangent to solve problems.
[CN, PS, T, V] [ICT : C6–4.1, C6–4.3]

Notes:

- Students should write the sinusoidal function in the form $y = a\sin\left[b(x-c)\right] + d$ or $y = a\cos\left[b(x-c)\right] + d$.
- Transformations of tangent graphs are beyond the scope of this outcome.
- Graphing reciprocal trigonometric functions is beyond the scope of this outcome.
- Analyzing the characteristics of a sinusoidal function includes, but is not limited to, determining and describing amplitude, period, horizontal phase shift, vertical displacement, intercepts, domain, and range.
- Analyzing the characteristics of a tangent function includes, but is not limited to, determining and describing period, asymptotes, intercepts, domain, and range.

(See examples 15-20.)

Specific Outcome 5

Solve, algebraically and graphically, first- and second-degree trigonometric equations with the domain expressed in degrees and radians. **[CN, PS, R, T, V] [ICT : C6–4.1, C6–4.4]**

Notes:

- Solving equations using single trigonometric identity substitutions should be limited to reciprocal, quotient, Pythagorean, double-angle identities, and sum or difference identities.
- Solving a double-angle equation algebraically will only be approached by identity substitution, resulting in the removal of the double-angle identity. All other multiple-angle equations are beyond the scope of this outcome.

(See examples 12 and 21-26.)

Specific Outcome 6

Prove trigonometric identities, using:

- reciprocal identities
- quotient identities
- Pythagorean identities
- sum or difference identities (restricted to sine, cosine and tangent)
- double-angle identities (restricted to sine, cosine and tangent) **[R, T, V] [ICT: C6–4.1, C6–4.4]**

Note:

- Students should understand that proving an identity is different from solving an equation.

(See examples 27-32.)

Specific Outcomes	Acceptable Standard The student can	Standard of Excellence The student can also
SO 1	• solve problems involving the radian measure of an angle as a ratio of the subtended arc to the radius of a circle • convert from radians to degrees and vice versa • solve problems involving arc length, radius, and angle measure in either radians or degrees • determine the measures, in degrees or radians, of all angles that are co-terminal with a given angle in standard position, within a specified domain	• solve multi-step problems based on the relationship $\theta = \frac{a}{r}$ (angle conversion and calculations involving radius and diameter are not considered steps)
SO 2	• determine the missing coordinate of a point $P(x, y)$ that lies on the unit circle	
SO 3	• determine the approximate values of trigonometric ratios of angles, θ, where $\theta \in R$ • determine the exact values of trigonometric ratios of special angles, θ, where $\theta \in R$ • determine the exact values of all the trigonometric ratios, given the value of one trigonometric ratio in a restricted domain or the coordinates of a point on the terminal arm of an angle in standard position • determine the measures of the angles, θ, in degrees or radians, given the value of a trigonometric ratio, where $0 \leq \theta < 2\pi$ or $0° \leq \theta < 360°$, or given a point on the terminal arm of an angle in standard position	

SO 4	• sketch the graphs of $y = \sin x$, $y = \cos x$, and $y = \tan x$, and analyze the characteristics of the graph	
	• describe the characteristics of sinusoidal functions of the form $y = a\sin[b(x - c)] + d$ and $y = a\cos[b(x - c)] + d$ and sketch the graph	• describe the characteristics of sinusoidal functions where the parameter b must be factored and sketch the graph
	• give partial explanations of the relationships between equation parameters and transformations of sinusoidal functions	• give full explanations of the relationships between equation parameters and transformations of sinusoidal functions
	• determine an equation for a sinusoidal curve, finding 1, 2, or 3 parameters, given the graph, the characteristics, or a real-world situation	• determine the complete equation, finding all 4 parameters, for a sinusoidal curve given the graph, the characteristics, or a real-world situation
	• provide a partial explanation of how the characteristics of the graph of a trigonometric function relate to the conditions in a contextual situation	• provide a complete explanation of how the characteristics of the graph of a trigonometric function relate to the conditions in a contextual situation
SO 5	• given a trigonometric equation, identify restrictions for the equation or non-permissible values for the variable in the domain $0 \le \theta < 2\pi$ or $0° \le \theta < 360°$	• given a trigonometric equation, identify restrictions for the equation or non-permissible values for the variable in the domain $\theta \in R$
	• determine, in a restricted domain, the graphical solution for any trigonometric equation	
	• algebraically determine the solution set of first-degree trigonometric equations within a restricted domain or the general solution	
	• in a restricted domain within $0 \le \theta < 2\pi$ or $0° \le \theta < 360°$, algebraically determine the solution set of second-degree trigonometric equations	• in a restricted domain outside $0 \le \theta < 2\pi$ or $0° \le \theta < 360°$, algebraically determine the solution set of second-degree trigonometric equations

		• algebraically determine, in a restricted domain, the solution set of trigonometric equations involving sum and difference, double-angle, or Pythagorean identity substitutions • determine the general solution of – second-degree trigonometric equations – trigonometric equations involving sum and difference, double-angle, or Pythagorean identity substitutions
SO 6	• explain the difference between a trigonometric identity and a trigonometric equation • explain the difference between verifying for a given value and proving an identity for all permissible values	
	• verify a trigonometric identity graphically or numerically for a given value	• determine the non-permissible values of a trigonometric identity
	• algebraically simplify and prove simple identities and recognize that there may be non-permissible values	• algebraically simplify and prove more-difficult identities that include sum and difference identities, double-angle identities, conjugates, or the extensive use of rational operations
	• determine the exact value of a trigonometric ratio using the sum, difference, and double-angle identities of sine and cosine	• determine the exact value of a trigonometric ratio using the sum, difference, and double-angle identities of a tangent
SO 1–SO 6	• participate in and contribute to the problem-solving process for problems that require analysis of trigonometry studied in Mathematics 30–1	• complete the solutions to problems that require the analysis of trigonometry studied in Mathematics 30–1

Examples

Students who achieve the acceptable standard should be able to answer all of the following questions, except for any part of a question labelled ***SE***. Parts labelled ***SE*** are appropriate examples for students who achieve the standard of excellence.

Note: *In the multiple-choice questions that follow, * indicates the correct answer. Please be aware that the worked solutions show possible strategies; there may be other strategies that could be used.*

SO 1

1. An angle, in radians, that is co-terminal with 30° is

A. $-\frac{5\pi}{6}$

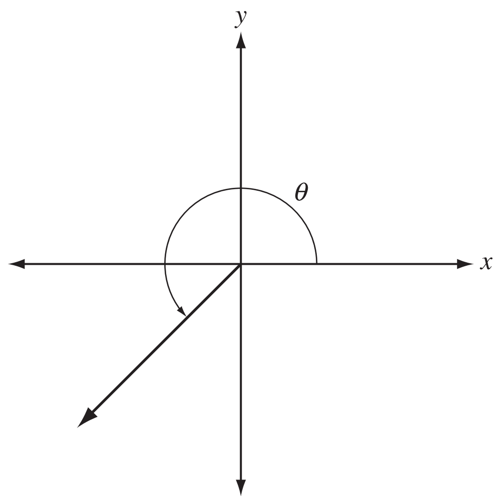
B. $-\frac{13\pi}{6}$

C. $\frac{7\pi}{6}$

***D.** $\frac{25\pi}{6}$

Use the following information to answer question 2.

An angle, θ, in standard position, is shown below.



SO 1

2. The best estimate of the rotation angle θ is

A. 1.25 radians

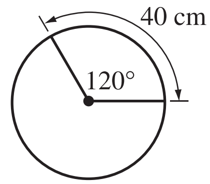
B. 3.12 radians

***C.** 4.01 radians

D. 5.38 radians

Use the following information to answer numerical-response question 3.

Mary is given the diagram below, showing an angle rotation of 120°. The arc length of the sector is 40 cm.



Statement 1 The radius of the circle, to the nearest centimetre, is 19 cm.

Statement 2 An equivalent angle rotation is $\frac{4\pi}{3}$.

Statement 3 If the arc length on this circle increases to 80 cm, then the central angle must be 240°.

Statement 4 Mary can determine the radius of the circle by dividing the given angle by the arc length.

Numerical Response

SO 1 **3.** The two statements above that are correct are numbered _____ and _____.

Solution: 13 or 31

Statement 1: Correct

$$\theta = \frac{a}{r}$$

$$r = \frac{40}{\left(120° \times \frac{\pi}{180°}\right)}$$

$$r \approx 19 \text{ cm}$$

Statement 2: Incorrect

$$120° \times \frac{\pi}{180°} = \frac{2\pi}{3}$$

Statement 3: Correct

$$\theta = \frac{a}{r}$$

$$\theta = \frac{80}{19} \times \frac{180°}{\pi}$$

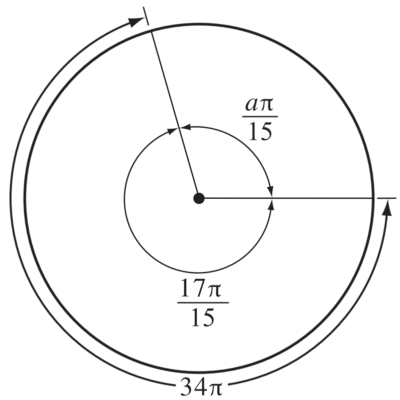
$$\theta = 240°$$

Statement 4: Incorrect

The radius of a circle is determined by dividing the arc length by the given angle.

Use the following information to answer numerical-response questions 4 and 5.

A circle with a radius r, an indicated arc length of 34π, and two central angles of $\frac{a\pi}{15}$ and $\frac{17\pi}{15}$ is shown below.



Numerical Response

SO 1 **4.** For the angle $\frac{a\pi}{15}$, the value of $\boldsymbol{a}$ is ______.

Solution: 13

$$\frac{a\pi}{15} + \frac{17\pi}{15} = 2\pi$$

$$\frac{a\pi}{15} = 2\pi - \frac{17\pi}{15}$$

$$\frac{a\pi}{15} = \frac{13\pi}{15}$$

Therefore, $a = 13$.

Numerical Response

SO 1 **5.** The length of the radius, ***r***, of the circle, to the nearest whole number, is _____.

Solution: 30

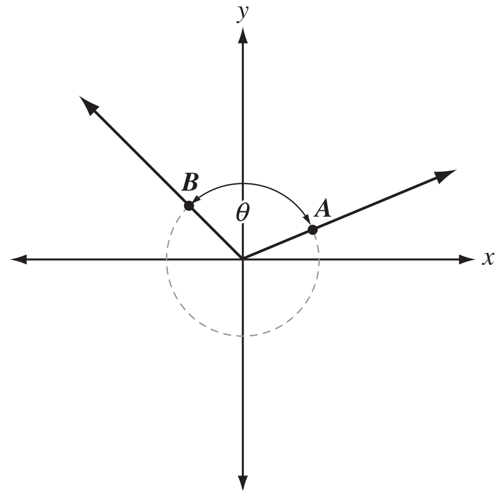
$$\theta = \frac{a}{r}$$

$$r = \frac{34\pi}{\frac{17\pi}{15}}$$

Therefore, $r = 30$.

Use the following information to answer numerical-response question 6.

Point $A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ and Point $B\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ lie on the terminal arm of two different angles in standard position on the circle. The angle, θ, where $0 < \theta < \pi$, can be expressed in the form $\frac{m\pi}{n}$.



Numerical Response

SO 1
SO 2

6. The values of ***m*** and ***n*** are, respectively, ______ and ______.

Solution: In standard position, angle $\frac{\pi}{6}$ has a terminal arm that passes through Point $A\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ and the angle $\frac{3\pi}{4}$ has a terminal arm that passes through Point $B\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$.

$$\theta = \frac{3\pi}{4} - \frac{\pi}{6}$$

$$\theta = \frac{7\pi}{12}$$

$m = 7$ and $n = 12$

Use the following information to answer numerical-response question 7.

If the point $P(0.2, k)$ lies on a circle with a centre at the origin and a radius of 1, then the exact value of k can be expressed as $\pm\sqrt{b}$.

Numerical Response

SO 2

7. The value of ***b***, to the nearest hundredth, is __________.

Solution: 0.96

$$x^2 + y^2 = r^2$$
$$(0.2)^2 + k^2 = (1)^2$$
$$k = \pm\sqrt{0.96}$$
$$b = 0.96$$

SO 2

8. The terminal arm of θ, when drawn in standard position, contains the point $P(x, y)$ where P is on the unit circle. If $\sin\theta = \frac{7}{10}$ and $\tan\theta < 0$, then what is the value of x?

***A.** $-\frac{\sqrt{51}}{10}$

B. $\frac{\sqrt{51}}{10}$

C. $\frac{3}{10}$

D. $-\frac{3}{10}$

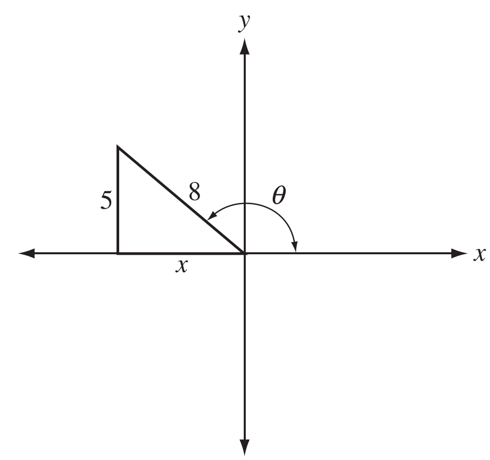
SO 2
SO 3

9. On a unit circle, Point $P\left(-\frac{5}{13}, \frac{12}{13}\right)$ lies on the terminal arm of angle θ in standard position. What are the exact values of the 6 trigonometric ratios for angle θ?

Possible solution: Since Point P is on the unit circle, the x-coordinate is the ratio for cosine, the y-coordinate is the ratio for sine, and $\frac{y}{x}$ is the ratio for tangent.

$$\therefore \sin\theta = \frac{12}{13}; \cos\theta = -\frac{5}{13}; \tan\theta = -\frac{12}{5}; \csc\theta = \frac{13}{12}; \sec\theta = -\frac{13}{5}; \cot\theta = -\frac{5}{12}$$

Use the following information to answer question 10.



SO 3

10. Given that $\csc\theta = \frac{8}{5}$, where $\frac{\pi}{2} < \theta < \pi$, determine the **exact** value of $\tan\theta$.

Possible solution: $x^2 + (5)^2 = 8^2$

$$x^2 = 39$$

$$x = \pm\sqrt{39}$$

Since θ is a second quadrant angle, $x = -\sqrt{39}$.

Therefore, $\tan\theta = -\frac{5}{\sqrt{39}}$ or $\tan\theta = -\frac{5\sqrt{39}}{39}$.

SO 3

11. The **exact** value of $\sin\left(-\frac{\pi}{6}\right) + \cos\left(\frac{7\pi}{4}\right)$ is

***A.** $\frac{\sqrt{2}-1}{2}$

B. $\frac{\sqrt{2}+1}{2}$

C. $\frac{\sqrt{3}+\sqrt{2}}{2}$

D. $\frac{\sqrt{3}-\sqrt{2}}{2}$

SO 3
SO 5

12. If $\tan\theta = \frac{5}{2}$, where $0 \le \theta < 2\pi$, then the largest positive value of θ, to the nearest tenth, is __________ rad.

Possible solution: $\theta = \tan^{-1}\left(\frac{5}{2}\right)$

$\theta \approx 1.19$

Tangent is positive in quadrants I and III; therefore, the largest positive value for the angle in the given domain is

$\theta = \pi + 1.19$

$\theta \approx 4.3$

Use the following information to answer numerical-response question 13.

Each of the trigonometric ratios listed below results in a value of zero or is undefined.

$\tan\left(\frac{\pi}{2}\right)$

$\cot\left(\frac{3\pi}{2}\right)$

$\sin\pi$

$\csc(2\pi)$

Numerical Response

SO 3

13. Use the following code to indicate that the value of the ratio is zero or that the ratio is undefined.

1 = The value of the ratio is zero.
2 = The ratio is undefined.

	______	______	______	______
Ratio:	$\tan\left(\frac{\pi}{2}\right)$	$\cot\left(\frac{3\pi}{2}\right)$	$\sin\pi$	$\csc(2\pi)$

Solution: 2112

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

Use the following information to answer numerical-response question 14.

For the angles $\frac{\pi}{6}$, $\frac{5\pi}{6}$, $\frac{7\pi}{6}$, and $\frac{11\pi}{6}$, the following statements are given.

Statement 1 These angles in degrees are, respectively, 30°, 150°, 210°, and 300°.

Statement 2 They are all part of the general solution $\theta = \frac{\pi}{6} + 2n\pi, n \in I$.

Statement 3 The values of $\sin\left(\frac{7\pi}{6}\right)$ and $\cos\left(\frac{5\pi}{6}\right)$ are both negative.

Numerical Response

SO 1
SO 3

14. The statement that is true from the list above is number _____.

Solution: 3

Statement 1: False

$$\frac{11\pi}{6} \times \frac{180°}{\pi} = 330°$$

Statement 2: False

The general solution $\theta = \frac{\pi}{6} + 2n\pi, n \in I$ does not include $\frac{5\pi}{6}$, $\frac{7\pi}{6}$, and $\frac{11\pi}{6}$.

Statement 3: True

Sine is negative in Quadrant III
and cosine is negative in Quadrant II.

Numerical Response

SO 4

15. For the function $y = a\cos\theta + d$, where the range is [–4, 10], the values of $\boldsymbol{a}$ and $\boldsymbol{d}$ are, respectively, _____ and _____.

Solution: $a = 7$ and $d = 3$

Since the maximum value is 10 and the minimum value is –4, the amplitude is $\frac{10-(-4)}{2}$.

The vertical displacement is maximum – amplitude = 10 – 7.

SE

SO 4

16. *A function is represented by the equation* $y = \sin(3x + \pi) + 7$. *The numerical value of the phase shift, when compared to* $y = \sin x$, *is* ___***i***___ *and the period of the corresponding graph is* ___***ii***___ .

The statement above is completed by the information in row

Row	***i***	***ii***
A.	π	3
B.	π	$\frac{2\pi}{3}$
***C.**	$\frac{\pi}{3}$	$\frac{2\pi}{3}$
D.	$\frac{\pi}{3}$	3

Note: This item is SE since the *b* value must be factored out in order to identify the phase shift.

Numerical Response

SO 4

17. Given that $f(\theta) = \cos(n\theta)$ has the same period as the graph of $g(\theta) = \tan\theta$, the value of ***n*** is __________.

Solution: 2

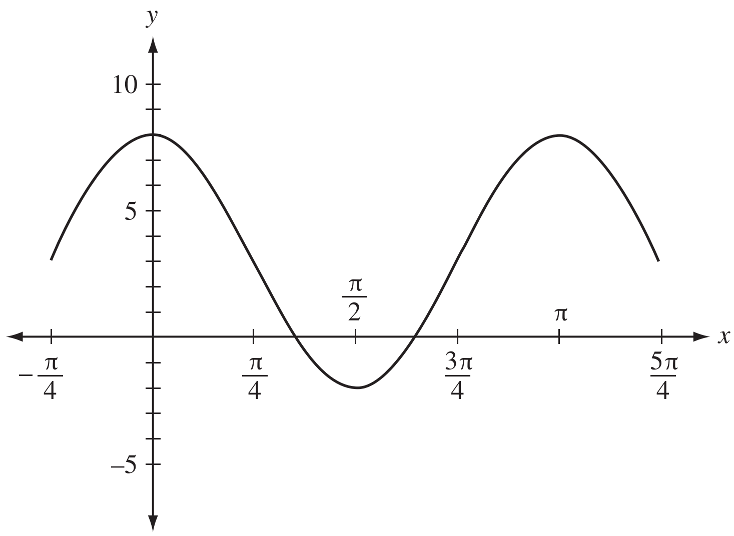
Period of $g(\theta) = \tan\theta$ is π.

Period of $f(\theta) = \cos(n\theta)$ is $\frac{2\pi}{n}$.

In order for the two functions to have the same period of π, $n = 2$.

Use the following information to answer numerical-response question 18.

The partial graph of the cosine function below has a minimum point at $\left(\frac{\pi}{2}, -2\right)$ and a maximum point at $(\pi, 8)$ as shown below. The equation of the function can be expressed in the form $y = a\cos\left(b(x - c)\right) + d,\ a, b, c, d \in W$.



Numerical Response

SE

SO 4

18. With a minimum possible phase shift, the values of ***a***, ***b***, ***c***, and ***d*** are, respectively, _____, _____, _____, and _____.

Solution: $a = \dfrac{8 - (-2)}{2} = 5$

$b = \dfrac{2\pi}{\text{period}} = \dfrac{2\pi}{\pi} = 2$

$c = 0$

$d = 8 - 5 = 3$

$\therefore$ solution is 5203

Note: This item is SE since all 4 parameters must be determined.

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

Use the following information to answer question 19.

For the graph of the function $f(x) = -3\sin\left[2(x-5)\right] + d$, the following statements were made.

Statement 1 The amplitude is 3.

Statement 2 The maximum value is $(d - 3)$.

Statement 3 The period is 2π.

Statement 4 When compared to the graph of $g(x) = -3\sin(2x) + d$, the graph of $y = f(x)$ has been horizontally translated 5 units to the right.

Statement 5 If $d > 3$, then the graph of $y = f(x)$ will have no x-intercepts.

SO 4

19. The true statements are

A. 1, 5 only
B. 2, 3 only
***C.** 1, 4, 5 only
D. 2, 4, 5 only

Possible solution:

Statement 1: True

The value of $|a|$ identifies the amplitude of the function.

Statement 2: False

The value of d identifies the vertical displacement. The maximum value is $(d + 3)$.

Statement 3: False

The period is $\frac{2\pi}{2} = \pi$.

Statement 4: True

The value of c identifies the phase shift.

Statement 5: True

If the vertical displacement is greater than the amplitude, then the graph will not touch or cross the x-axis.

Use the following information to answer question 20.

> The height of a point on a Ferris wheel, h, in metres above the ground, as a function of time, t, in seconds can be represented by a sinusoidal function. The maximum height of the Ferris wheel above the ground is 17 m and the minimum height is 1 m. It takes the Ferris wheel 60 seconds to complete two full rotations.

SE

SO 4

20. Assuming that the particular point starts at the minimum height above the ground, write an equation for the height of this point on the Ferris wheel, h, as a function of time, t, in the form $h = a\cos[b(t-c)] + d$.

Possible solution: Amplitude is 8 m. Period is 30 s.

$$a = \frac{17-1}{2} = 8$$

$$b = \frac{2\pi}{30} = \frac{\pi}{15}$$

$$c = 15 \text{ s to the right or to the left} = \pm 15$$

$$d = \frac{17+1}{2} = 9$$

$$\therefore h = 8\cos\left[\frac{\pi}{15}(t-15)\right] + 9 \text{ or } h = 8\cos\left[\frac{\pi}{15}(t+15)\right] + 9$$

Note: This item is SE since all 4 parameters of the function must be determined.

SO 5

21. The values of θ, where $180° \leq \theta < 360°$, in the equation $2\cos^2\theta + \cos\theta = 0$, are

A. $\theta = 90°, 120°$

***B.** $\theta = 240°, 270°$

C. $\theta = 60°, 90°, 270°, 300°$

D. $\theta = 90°, 120°, 240°, 270°$

Use the following information to answer question 22.

An incorrect solution to the equation $\cos\theta = \dfrac{\cos\theta}{\csc\theta}$, where $0° \le \theta < 360°$, is shown below.

Step 1 $\cos\theta = \cos\theta\left(\dfrac{1}{\csc\theta}\right)$

Step 2 $1 = \dfrac{1}{\csc\theta}$

Step 3 $\csc\theta = 1$

Step 4 $\cos\theta = 1$

Step 5 $\theta = 0°, 180°$

SE

SO 5

22. The step that records the **first** mistake in the solution is

***A.** Step 2
B. Step 3
C. Step 4
D. Step 5

SE

SO 5

23. Determine a general solution of $\cot^2\theta - 1 = 0$, expressed in radians.

Possible solution: $\cot^2\theta = 1$

$\cot\theta = \pm 1$

$\tan\theta = \pm 1$

$\theta = \dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}$, etc.

Therefore, the general solution is $\theta = \dfrac{\pi}{4} + \dfrac{n\pi}{2}, n \in I$.

Note: This item is SE since a general solution for a second-degree equation is required.

SE

SO 5

24. Determine the solution for the equation $2\cos^2 x + \sin x - 1 = 0$, where $-\pi \le x \le \pi$.

Possible solution: $2\cos^2 x + \sin x - 1 = 0$

$$2(1 - \sin^2 x) + \sin x - 1 = 0$$

$$2 - 2\sin^2 x + \sin x - 1 = 0$$

$$2\sin^2 x - \sin x - 1 = 0$$

$$(2\sin x + 1)(\sin x - 1) = 0$$

$$\sin x = -\frac{1}{2} \text{ or } \sin x = 1$$

$$x = -\frac{5\pi}{6},\ -\frac{\pi}{6},\ \frac{\pi}{2}$$

Note: This item is SE since it involves a Pythagorean identity substitution and since the restricted domain is outside $[0, 2\pi]$.

SE

SO 5

25. Graphically solve for θ, where $-180° \le \theta \le 0°$, given $\left(2 - \sqrt{3}\sec\theta\right)\left(\sec\theta + 3\right) = 0$. State answers to the nearest degree.

Possible solution: $\theta = -109°$ and $-30°$

Enter the following function into the calculator.

$$y_1 = \left(2 - \frac{\sqrt{3}}{\cos x}\right)\left(\frac{1}{\cos x} + 3\right)$$

A window that could be used is x: [–180, 0, 30], y: [–5, 5, 1].

The x-intercepts are the solutions to the original equation.

Use the following information to answer question 26.

A Mathematics 30–1 class was asked to determine a general solution to the equation $2\sin\theta\cos\theta - \cos\theta = 0$, in degrees. The answers provided by four different students are shown below.

Student 1 $\theta = 60° + n(120°),\ n \in I$

Student 2 $\theta = 90° + n(360°),\ n \in I$, and $\theta = 30° + n(120°),\ n \in I$

Student 3 $\theta = n(180°)$, $\theta = 60° + n(360°)$, and $\theta = 300° + n(360°),\ n \in I$

Student 4 $\theta = 90° + n(180°)$, $\theta = 30° + n(360°)$, and $\theta = 150° + n(360°),\ n \in I$

SO 5

26. The two students who provided a correct general solution are numbered

A. 1 and 3
B. 1 and 4
C. 2 and 3
***D.** 2 and 4

Note: This item is SE since it involves determining a general solution for a second-degree equation.

SO 6 **27.** Three students were given the identity $\frac{\sin^2\theta - 1}{\cos\theta} = -\cos\theta$, where $\cos\theta \neq 0$.

a) Student A substituted $\theta = \frac{\pi}{3}$ into both sides of the equation and got LS = RS. Student B entered LS into y_1 and RS into y_2 and concluded that the graphs are exactly the same. Explain why these methods are not considered a proof of this identity.

Possible solution: Student A is verifying the identity using a single value of θ, whereas a proof is valid for all permissible values of θ. Student B is verifying that the graphs of each side of the identity are the same for all values of θ, but is only examining a limited portion of both graphs.

b) Student C correctly completed an algebraic process to show LS = RS. Show a process Student C might have used.

Possible solution: $\text{LS} = \frac{\sin^2\theta - 1}{\cos\theta}$ $\qquad$ $\text{RS} = -\cos\theta$

$$= \frac{-\cos^2\theta}{\cos\theta}$$

$$= -\cos\theta$$

$$\therefore \text{LS} = \text{RS}$$

SE **c)** Which non-permissible values of θ should be stated for this identity?

Possible solution: $\cos\theta \neq 0$

$$\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}, \text{etc.}$$

$$\theta \neq \frac{\pi}{2} + n\pi,\ n \in I$$

Note: This item is SE since since non-permissible values are required.

SE

SO 6

28. The expression $\dfrac{\cot x + \csc x}{\sec x + 1}$, for all permissible values of x, is equivalent to

A. $\sin x$

B. $\tan x$

C. $\csc x$

***D.** $\cot x$

Possible solution:

$$\frac{\dfrac{\cos x}{\sin x} + \dfrac{1}{\sin x}}{\dfrac{1}{\cos x} + 1} = \frac{\dfrac{\cos x + 1}{\sin x}}{\dfrac{1 + \cos x}{\cos x}}$$

$$= \frac{\cos x + 1}{\sin x} \times \frac{\cos x}{1 + \cos x}$$

$$= \frac{\cos x}{\sin x}$$

$$= \cot x$$

Note: This item is SE since it entails simplifying an identity involving rational operations.

Use the following information to answer numerical-response question 29.

Each trigonometric expression below can be simplified to a single numerical value.

1 $\sin x - \dfrac{\tan x}{\sec x}$

2 $\cot^2 x - \csc^2 x$

3 $\dfrac{1}{7}\cos^2 x + \dfrac{1}{7}\sin^2 x$

Numerical Response

SO 6

29. When the numerical values of the simplified expressions are arranged in ascending order, the expression numbers are _____, _____, and _____.

Solution: 213

1 $\sin x - \dfrac{\tan x}{\sec x} = \sin x - \dfrac{\sin x}{\cos x} \times \dfrac{\cos x}{1} = \sin x - \sin x = 0$

2 $\cot^2 x - \csc^2 x = -1$

3 $\dfrac{1}{7}\cos^2 x + \dfrac{1}{7}\sin^2 x = \dfrac{1}{7}\left(\cos^2 x + \sin^2 x\right) = \dfrac{1}{7}$

SE

SO 6

30. What is the **exact** value of $\tan 75°$?

Possible solution: $\sqrt{3} + 2$ or $\dfrac{\sqrt{3}+1}{\sqrt{3}-1}$

$$\begin{aligned}
\tan 75° &= \tan(45° + 30°) \\
&= \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°} \\
&= \frac{1 + \frac{1}{\sqrt{3}}}{1 - 1\left(\frac{1}{\sqrt{3}}\right)} \\
&= \frac{\frac{\sqrt{3}+1}{\sqrt{3}}}{\frac{\sqrt{3}-1}{\sqrt{3}}} \\
&= \frac{\sqrt{3}+1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}-1} \\
&= \frac{\sqrt{3}+1}{\sqrt{3}-1} \text{ or } \sqrt{3} + 2 \text{ (rationalize the denominator)}
\end{aligned}$$

Note: This item is SE since it involves the sum identity of a tangent.

SE

SO 6

31. Prove algebraically that $\dfrac{2\tan x}{1 - \tan^2 x} = \dfrac{\sin(2x)}{\cos^2 x - \sin^2 x}$, where $x \neq \dfrac{\pi}{4} + \dfrac{n\pi}{2}$, $n \in I$.

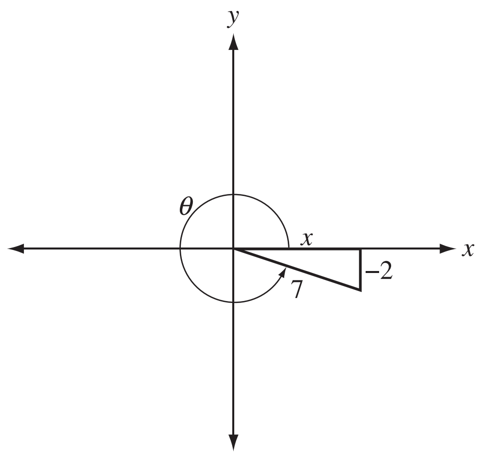
Possible solution:

Left Side	**Right Side**
$\dfrac{2\tan x}{1 - \tan^2 x}$	$\dfrac{\sin(2x)}{\cos^2 x - \sin^2 x}$
$\tan(2x)$	$\dfrac{\sin(2x)}{\cos(2x)}$
	$\tan(2x)$

LS = RS

Note: This item is SE since it involves a double-angle tangent identity.

Use the following information to answer question 32.



SO 3
SO 6

32. Given that $\sin\theta = -\frac{2}{7}$ and $\cot\theta < 0$, determine the **exact** value of $\cos\left(\theta - \frac{2\pi}{3}\right)$.

Possible solution: $x^2 + (-2)^2 = 7^2$

$$x^2 = 45$$

$$x = \pm\sqrt{45}$$

$$x = \pm 3\sqrt{5}$$

Since θ is in Quadrant IV, $x = 3\sqrt{5}$ and $\cos\theta = \frac{3\sqrt{5}}{7}$.

$$\cos\left(\theta - \frac{2\pi}{3}\right) = \cos\theta\cos\frac{2\pi}{3} + \sin\theta\sin\frac{2\pi}{3}$$

$$= \left(\frac{3\sqrt{5}}{7}\right)\left(-\frac{1}{2}\right) + \left(-\frac{2}{7}\right)\left(\frac{\sqrt{3}}{2}\right)$$

$$= \frac{-3\sqrt{5}}{14} - \frac{2\sqrt{3}}{14}$$

$$= \frac{-3\sqrt{5} - 2\sqrt{3}}{14}$$

Permutations, Combinations, and Binomial Theorem

General Outcome

Develop algebraic and numeric reasoning that involves combinatorics.

General notes:

- Students are expected to simplify expressions involving ${}_nP_r$, ${}_nC_r$, and $n!$ algebraically.
- Probability is beyond the scope of this course.

Specific Outcomes

Specific Outcome 1

Apply the fundamental counting principle to solve problems. **[C, PS, R, V] [ICT: C6–2.3]**

Note:

- Alternative methods of problem solving involving pictorial or visual explanations (i.e. tree diagrams, lists) are appropriate.

(See examples 1-3.)

Specific Outcome 2

Determine the number of permutations of *n* elements taken *r* at a time to solve problems.
[C, PS, R, V]

Notes:

- Permutation problems could involve repetition of like elements and constraints.
- Circular and ring permutations are beyond the scope of this outcome.
- Single 2-dimensional pathways may be used as an application of repetition of like elements.

(See examples 4-7.)

Specific Outcome 3

Determine the number of combinations of n different elements taken r at a time to solve problems.
[C, PS, R, V]

Note:

- Students are expected to know both ${}_nC_r$ and $\binom{n}{r}$ notation.

(See examples 8-10.)

Specific Outcome 4

Expand powers of a binomial in a variety of ways, including using the binomial theorem (restricted to exponents that are natural numbers). **[CN, R, V]**

Notes:

- Teachers may choose to show the relationship between the rows of Pascal's triangle and the numerical coefficients of the terms in the expansion of a binomial $(x + y)^n, n \in N$.
- Students are expected to recognize various patterns in the binomial expansion.

(See examples 11-15.)

Specific Outcomes	**Acceptable Standard** The student can	**Standard of Excellence** The student can also
SO 1	• apply the fundamental counting principle to various problems involving at most two cases or constraints • understand and use factorial notation	• apply the fundamental counting principle to various problems involving three or more cases or constraints
SO 2 SO 3	• solve problems involving the terms *and* or *or*	• solve problems involving the terms *at least* or *at most* • solve problems involving cases where items cannot be together
	• solve problems involving permutations or combinations	• solve problems involving both permutations and combinations
	• solve problems involving permutations when two or more elements are identical (repetitions), with at most one constraint	• solve problems involving permutations when two or more elements are identical (repetitions), with more than one constraint
	• solve for n in equations involving one occurrence of ${}_nP_r$ or ${}_nC_r$ given r, where $r \leq 3$, and identify extraneous solutions	
	• determine solutions to problems involving at most two cases or constraints	• determine solutions to problems involving three or more cases or constraints
SO 4	• solve problems involving patterns that exist in the binomial expansion of $(x + y)^n$	
	• expand $(x + y)^n$ or determine a specified term in the expansion of a binomial with linear terms	• expand $(x + y)^n$ or determine a specified term in the expansion of a binomial with non-linear terms • determine x or y in $(x + y)^n$ given a specified term in its expansion
	• provide a partial solution and explain simple mathematical strategies in a problem-solving context involving combinatorics studied in Mathematics 30-1	• provide a complete solution and explain complex mathematical strategies in a problem-solving context involving combinatorics studied in Mathematics 30-1

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

Examples

Students who achieve the acceptable standard should be able to answer all of the following questions, except for any part of a question labelled ***SE***. Parts labelled ***SE*** are appropriate examples for students who achieve the standard of excellence.

Note: *Please be aware that the worked solutions show possible strategies; there may be other strategies that could be used. In the multiple-choice questions that follow, * indicates the correct answer.*

SO 1

1. If all of the letters in the word DIPLOMA are used, then how many different arrangements are possible that begin and end with an I, O, or A?

A. $3 \times 5^5 \times 3$

B. $3 \times 5^5 \times 2$

C. $3 \times 5! \times 3$

***D.** $3 \times 5! \times 2$

Possible solution: $3 \times 5 \times 4 \times 3 \times 2 \times 1 \times 2 = 720$

Possible solution: ${}_3P_2 \times {}_5P_5 = 720$

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

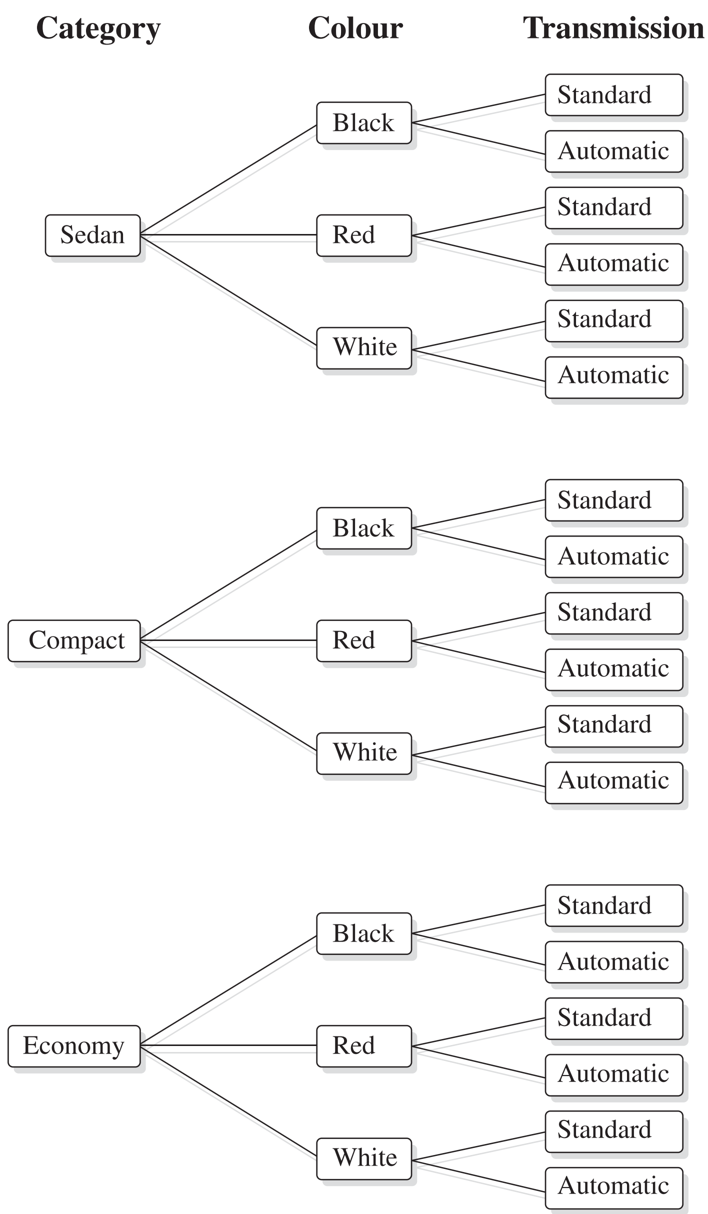
Use the following information to answer numerical-response question 2.

Josh wants to rent a car. He has narrowed his choices to a sedan, a compact, or an economy car. The colours available are black, red, or white. He may also choose between a standard and an automatic transmission.

Numerical Response

SO 1 **2.** The number of options that Josh has is _____.

Solution: 18



To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

Use the following information to answer numerical-response question 3.

The identification code on a bank card consists of 1 digit followed by 2 letters. The code must meet the following conditions:

- The digit must be odd.
- The letters A, E, I, O, and U cannot be used.
- Letters cannot be used more than once.

Numerical Response

SO 1 **3.** The number of possible identification codes that can be created with these conditions is _____.

Solution: $5 \times 21 \times 20 = 2\ 100$

Use the following information to answer question 4.

A volleyball team made up of 6 players stands in a line facing the camera for a picture.

SO 2 **4.** If Joan and Emily must be together, then an expression that represents the number of different arrangements for the picture is

***A.** $2! \times {}_5P_5$

B. $2! \times {}_5C_5$

C. $2! \times {}_6P_6$

D. $2! \times {}_6C_6$

SE

SO 2

5. Determine the number of different arrangements using all the letters of the word ACCESSES that

a) begin with at least two S's
b) begin with exactly two S's
c) Explain why the answers in questions a) and b) are different.

Possible solution: a) $\dfrac{3 \times 2 \times 5 \times 5!}{3!2!2!} + \dfrac{3 \times 2 \times 1 \times 5!}{3!2!2!} = 150 + 30 = 180$

b) $\dfrac{3 \times 2 \times 5 \times 5!}{3!2!2!} = 150$

c) The answers are different because the third letter cannot be an S in part b).

Possible solution: a) The arrangement must begin with two S's.
Therefore, we only need to arrange the remaining letters: AEECCS

$\dfrac{6!}{2!2!} = 180$

b) Arrange the letters AEECCS as in part a) above.
Then subtract the arrangements that begin with a third S.

$\dfrac{6!}{2!2!} - \dfrac{5!}{2!2!} = 150$

c) The answers are different because the third letter cannot be an S in part b).

Note: This item is SE since it involves problems using the term *at least*.

Use the following information to answer numerical-response question 6.

At a car dealership, the manager wants to line up 10 cars of the same model in the parking lot. There are 3 red cars, 2 blue cars, and 5 green cars.

Numerical Response

SE

SO 2

6. If all 10 cars are lined up in a row facing forward and if the blue cars **cannot** be together, then the number of possible arrangements is _____.

Possible solution: $\underbrace{\frac{10!}{3!2!5!}}_{\text{Total possible arrangements}} - \underbrace{\frac{9!}{3!5!}}_{\text{Arrangements with blue together}} = 2\,016$

Possible solution: $\frac{8! \times {}_9P_2}{3!2!5!} = 2\,016$

Note: This item is SE since it involves a scenario in which items cannot be together.

Use the following information to answer numerical-response question 7.

If 14 different types of fruit are available, how many different fruit salads could be made using exactly 5 types of fruit?

Student 1	Kevin used $\frac{14!}{5!}$ to solve the problem.
Student 2	Ron suggested using ${}_{14}P_5$.
Student 3	Michelle solved the problem using ${}_{14}C_9$.
Student 4	Jackie thought that 5! would give the correct answer.
Student 5	Stan decided to use $\binom{14}{5}$.

Numerical Response

SO 2

7. The correct solution would be obtained by student number _____ and student number _____.

Solution: 35 or 53

The order of fruits being added to the salad does not make a difference; therefore, this must be a combination. Both Michelle and Stan are correct since ${}_{14}C_5 = {}_{14}C_9$. Choosing 5 fruits from the 14 available is the same as not choosing 9 fruits from the 14 available.

Use the following information to answer numerical-response question 8.

At a meeting, every person shakes hands with every other person exactly once.

Numerical Response

SO 3

8. If there are 36 handshakes in total, then the number of people at the meeting is _____.

Solution: 9

$$
\begin{aligned}
{}_nC_2 &= 36 \\
\frac{n!}{(n-2)!2!} &= 36 \\
n(n-1) &= 72 \\
n^2 - n - 72 &= 0 \\
(n-9)(n+8) &= 0 \\
n &= 9, \cancel{-8}
\end{aligned}
$$

There were 9 people at the meeting.

SO 3

9. A 6-person health council must be created from a group of 8 doctors and 40 nurses. The expression that represents the number of different councils that contain at most 2 doctors is

A. $({}_{40}P_6) + ({}_8P_1)({}_{40}P_5) + ({}_8P_2)({}_{40}P_4)$

***B.** $({}_{40}C_6) + ({}_8C_1)({}_{40}C_5) + ({}_8C_2)({}_{40}C_4)$

C. $({}_{40}P_5)({}_8P_1) + ({}_{40}P_4)({}_8P_2)$

D. $({}_{40}C_5)({}_8C_1) + ({}_{40}C_4)({}_8C_2)$

Numerical Response

SE

SO 3

10. The number of different 4-letter arrangements possible using any 2 letters from the word SMILE and any 2 letters from the word FROG is _____.

Solution: $({}_5C_2 \times {}_4C_2) \times 4! = 1\,440$

Choose 2 letters from each word first, and then arrange the 4 letters.

Note: This item is SE since it involves both a combination and a permutation.

SO 4 **11.** The expanded form of $(2x + 3)^{(a-5)}$ has 7 terms. The value of ***a*** is ___*i*___ and the coefficient on the first term is ___*ii*___ .

The statement above is completed by the information in row

Row	*i*	*ii*
***A.**	11	64
B.	11	128
C.	12	64
D.	12	128

To go back after using an internal link, simultaneously press and hold Alt ← (in some browsers).

Use the following information to answer numerical-response question 12.

A student made the following statements regarding the expansion of $(a + b)^4$, written in descending powers of a.

Statement 1	The total number of terms is 5.
Statement 2	The sum of the coefficients of all the terms is 14.
Statement 3	For the term $4a^3b^m$, the value of m is 1.
Statement 4	The coefficient of the first term is ${}_4C_1$.

Numerical Response

SO 4 **12.** The two statements above that are true are numbered _____ and _____.

Solution: 13 (list in any order)

According to the binomial theorem, the terms of the binomial expansion will be:

$$(a + b)^4 = {}_4C_0(a^4) + {}_4C_1(a^3b^1) + {}_4C_2(a^2b^2) + {}_4C_3(a^1b^3) + {}_4C_4(b^4)$$

$$= a^4 + 4a^3b + 6a^2b^2 + 4ab^3 + b^4$$

Statement 1: True

The number of terms in the expansion of $(a + b)^n$ is $n + 1$; therefore, there are 5 terms.

Statement 2: False

The sum of all the coefficients in the expansion of $(a + b)^n$ is 2^n; therefore, the sum is $2^4 = 16$.

Statement 3: True

The sum of the exponents in each term must equal n; therefore, $m = 1$.

Statement 4: False

The coefficient of the first term is ${}_4C_0$.

SE

SO 4

13. In the expansion of $(3a - b^2)^{10}$, what is the coefficient of the term containing a^4b^{12}?

A. –17 010

B. –630

C. 630

***D.** 17 010

Note: This item is SE since the binomial contains non-linear terms.

Numerical Response

SE

SO 4

14. In the expansion of the binomial $\left(2a + \frac{1}{a}\right)^8$, the constant term is _____.

Solution: 1 120

The exponent of the variable for a constant term must be zero; i.e., a^0.

$${}_8C_4(2a)^4\left(\frac{1}{a}\right)^4$$

$$70\left(16a^4\right)\left(\frac{1}{a^4}\right)$$

1 120

Therefore, the constant term is 1 120.

Note: This item is SE since the binomial contains non-linear terms.

Numerical Response

SE

SO 4

15. A term in the expansion of $(ax - y)^6$ is $-252xy^5$. The numerical value of $\boldsymbol{a}$ is _____.

Solution: 42

Using the general term, $k = 5$,

$$\begin{aligned} t_{5+1} &= {}_6C_5(ax)^{6-5}(-y)^5 \\ -252xy^5 &= 6(ax)(-y^5) \\ -252 &= -6a \\ 42 &= a \end{aligned}$$

Note: This item is SE since it requires a parameter in the binomial to be found, given a specified term.

Website Links

Publication/Resource
Mathematics 10–12 Program of Studies *Mathematics 30–1 Information Bulletin* Using Calculators *General Information Bulletin* Quest A+ Mathematics Directing Words FAQs About Alberta's Math Program

Please note that if you cannot access one of the direct website links referred to in this document, you can find Diploma Examination-related materials on the Alberta Education website.