Released Items
Mathematics 30–2

Diploma Examinations Program 2019

Alberta

This document was written primarily for:

Students	✓	
Teachers	✓	of Mathematics 30–2
Administrators	✓	
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General Audience		
Others		

Alberta Education, Government of Alberta

2019–2020

Mathematics 30–2 Released Items

Distribution: This document is posted on the Alberta Education website.

Contents

Please note that if you cannot access one of the direct website links referred to in this document, you can find diploma examination-related materials on the Alberta Education website.

Introduction

The questions in this booklet are from the *January 2019 Mathematics 30–2 Diploma Examination*. Teachers may wish to use these questions in a variety of ways to help students develop and demonstrate an understanding of the concepts described in the Mathematics 30–2 Program of Studies. This material, along with the *Program of Studies*, *Information Bulletin*, and *Assessment Standards and Exemplars*, can provide insights that assist you with decisions about instructional planning.

These questions are released in both English and French by the Provincial Assessment Sector.

Additional documents

The Provincial Assessment Sector supports the instruction of Mathematics 30–2 with the following documents available online.

- *Mathematics 30-2 Information Bulletin*

- *Mathematics 30-2 Assessment Standards and Exemplars*

- *Mathematics 30-2 Released Materials*

- *Mathematics 30-2 Written-Response Information*

- *School Reports and Instructional Group Reports* (for January and June diploma examinations)

Mathematics 30–2 Diploma Examination January 2019 – Item Information

The following tables give the results for the machine-scored and written-response questions released from the examination. For each question, the table also gives the correct response, the topic, the outcome, the cognitive level, and the assessment standard.

Topics		Cognitive Levels		Standards
LR	Logical Reasoning	C	Conceptual	Acceptable
PR	Probability	P	Procedural	Excellence
RF	Relations and Functions	PS	Problem Solving	

Question	Diff.*	Key	Topic	Outcome	Cognitive Level	Standard
NR1	83.5%	478	LR	1	PS	Acceptable
MC1	77.6%	D	LR	1	PS	Acceptable
NR2	80.8%	42	LR	1	PS	Acceptable
MC2	76.1%	A	LR	2	C	Acceptable
MC3	71.0%	C	LR	2	C	Excellence
MC4	75.9%	C	PR	1	P	Acceptable
NR3	63.0%	135	PR	2	C	Acceptable
MC5	51.9%	C	PR	3	P	Acceptable
MC6	59.7%	B	PR	4	PS	Acceptable
MC7	55.1%	B	PR	4	PS	Excellence
NR4	45.7%	525	PR	6	PS	Acceptable
MC8	44.5%	C	PR	5	PS	Acceptable
MC9	51.0%	A	PR	6	C	Excellence
MC10	69.2%	B	PR	5	P	Acceptable
MC11	71.2%	D	RF	1	P	Acceptable
NR5	69.0%	6226	RF	1	PS	Acceptable

| Question	Diff.*	Key	Topic	Outcome	Cognitive Level	Standard
MC12	59.8%	B	RF	2	C	Acceptable
MC13	68.1%	D	RF	2	P	Acceptable
NR6	58.6%	36, 47	RF	2	PS	Acceptable
MC14	66.3%	C	RF	3	P	Acceptable
MC15	43.9%	C	RF	3	C	Excellence
MC16	62.6%	A	RF	4	PS	Acceptable
NR7	56.6%	4.57	RF	4	P	Acceptable
MC17	64.6%	B	RF	4	P	Acceptable
MC18	70.0%	B	RF	5	C	Acceptable
MC19	62.9%	D	RF	6	C	Excellence
NR8	68.6%	2.2	RF	7	P	Acceptable
MC20	42.2%	A	RF	7	C	Acceptable
MC21	57.8%	C	RF	7	C	Excellence
MC22	43.6%	D	RF	8	C	Excellence
MC23	70.7%	B	RF	8	PS	Acceptable
MC24	61.3%	A	RF	8	PS	Acceptable

*Difficulty—percentage of students answering the question correctly

Question	Average Raw Score	Key	Topic	Outcome	Conceptual Level	Standard
WR1	2.9/7	See Sample Solution	PR	1, 2, 3	C, P, PS	Acceptable, Excellence
WR2	4.0/7	See Sample Solution	RF, LR	5, 6, 2	C, P, PS	Acceptable, Excellence

Use the following information to answer numerical-response question 1.

In a particular puzzle, the digits 1 through 9 are placed in nine circles arranged in a triangle. Each digit can be used only once. When the puzzle is completed, the digits on each side of the triangle must add to 17. A partially completed puzzle is shown below.

A

6 B

E 9

2 — 5 — D — C
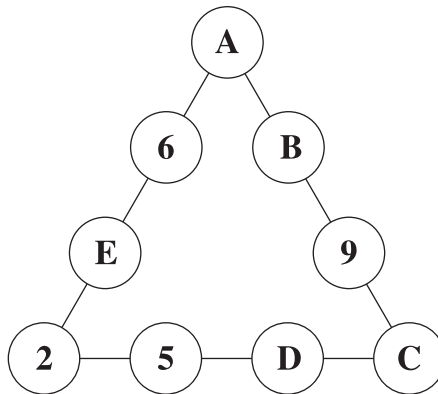
Numerical Response

1. When the puzzle is completed correctly, the value of

B is _____ (Record in the **first** column)

D is _____ (Record in the **second** column)

E is _____ (Record in the **third** column)

(Record your answer in the numerical-response section on the answer sheet.)

Shaye is playing a game where she is trying to determine a four-colour code that has been selected by her opponent. Colours may be repeated in the code. After each of Shaye's guesses, her opponent responds by providing information about how many colours are correct, and whether each colour is in the correct position or not. Shaye's first four guesses are shown below.

	Shaye's Guess	Opponent's Response
1	Red Red Red Red	**2** colours are correct and in the right positions
2	Red Red Blue Blue	**1** colour is correct and in the right position **1** colour is correct but in the wrong position
3	Red Yellow Red White	**3** colours are correct and in the right positions
4	Red Green Red White	**2** colours are correct and in the right positions **1** colour is correct but in the wrong position

**1.** The code selected by Shaye's opponent could be

A. Red Blue Red White

B. Red White Red Green

C. Red Yellow Red Yellow

D. Red Yellow Red Green

A puzzle consists of 16 squares arranged in a four-by-four grid. In the puzzle there are also four shapes that are outlined with bold black lines. The puzzle is completed by entering the digits from 1 through 4 into the squares so that each number appears exactly once in each row, column, and outlined shape. A partially completed puzzle is shown below.
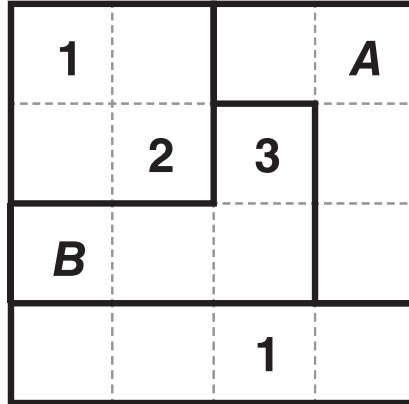
Numerical Response

2. When the puzzle above is completed correctly, the digit in the box marked

A is _____ (Record in the **first** column)

B is _____ (Record in the **second** column)

(Record your answer in the numerical-response section on the answer sheet.)

**Three Sets**

$X = \{BC, SK, ON, YT, NT\}$

$Y = \{AB, SK, MB, QC, ON\}$

$Z = \{BC, NS, NB, NU, NL\}$

2. Which of the following set operations will produce an empty set?

    A. Intersection of sets Y and Z
    B. Intersection of sets X and Y
    C. Union of sets Y and Z
    D. Union of sets X and Y

A class of 25 students was surveyed to determine the type of music they enjoy—alternative or pop. The results of the survey are illustrated in the partially completed Venn diagram below.

Legend

A Number of students who enjoy alternative music

P Number of students who enjoy pop music
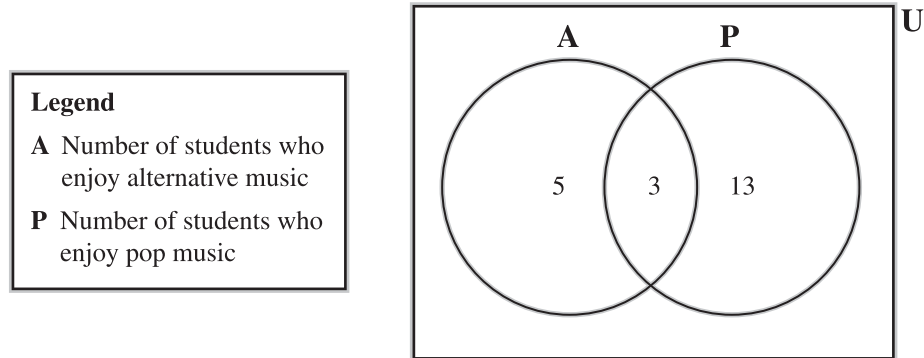
Frieda made the following statements about the sets above.

Statement 1	$A \subset P'$
Statement 2	$P \cup A = 21$
Statement 3	$P \cap A = 21$
Statement 4	$(P \cup A)' = 4$

3. The two statements above that are true are numbered

A. 1 and 2

B. 1 and 3

C. 2 and 4

D. 3 and 4

A bowl contains 5 white, 4 grey, and 3 patterned erasers, as shown below.
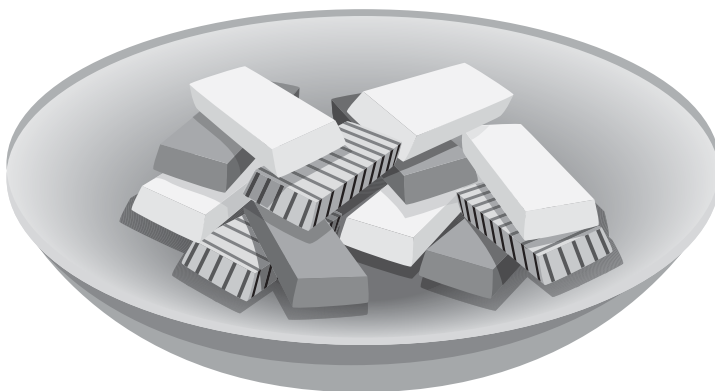
4. If an eraser is randomly selected from the bowl, then the odds against selecting a white eraser are

A. 7 : 12
B. 5 : 12
C. 7 : 5
D. 5 : 7

The probability that Soren watches the evening news on television on any given day is 0.63.

Numerical Response

3. Out of 365 days, the number of days that Soren is expected **not** to watch the evening news, to the nearest day, is _____ days.

(Record your answer in the numerical-response section on the answer sheet.)

A child is allowed to have 2 treats from a bag containing 21 chocolates, 11 hard candies, and 10 gummy candies. All three types of treat are in packages of similar size and shape. Since he cannot decide what he would like, the child reaches into the bag and randomly selects 2 treats, one at a time.

5. The probability that the child selects 1 chocolate and then 1 gummy candy, to the nearest thousandth, is

    A. 0.744

    B. 0.738

    C. 0.122

    D. 0.119

Use the following information to answer question 6.

Sally wants to decorate one wall in her room with movie posters. She will hang 5 of the 8 posters that she owns in a single row across the wall.

6. If Sally places her favourite movie poster in the middle, then the number of different poster arrangements that are possible is

    A. 120

    B. 840

    C. 1 680

    D. 6 720

From 1951 to 1981, the first digit of every three-digit telephone area code in North America was a number from 2 to 9, inclusive. The second digit was either 0 or 1. The last digit depended on the following rules.

- If the second digit was 0, the last digit could be any number from 1 to 9, inclusive.
- If the second digit was 1, the last digit could be any number from 2 to 9, inclusive.

Digits could be used more than once.

7. The number of different telephone area codes that could be formed with these restrictions was

    A.   128

    B.   136

    C.   144

    D.   153

Numerical Response

4. A golf shop sells 7 different drivers and 6 different irons. If Susan purchases 3 different drivers and 2 different irons, then the number of different golf club selections she can make is _____ .

(Record your answer in the numerical-response section on the answer sheet.)

Use the following information to answer question 8.

Finley has 8 game pieces that differ only in colour. Each game piece is either all red or all black. When Finley lines up the 8 game pieces in a single row, there are 28 distinguishable arrangements.

8. *Based on the information above, Finley could have ___i___ red game pieces and ___ii___ black game pieces.*

The statement above can be completed by the information in row

Row	i	ii
A.	4	4
B.	5	3
C.	6	2
D.	7	1

Use the following information to answer question 9.

A group of camp employees, 5 leaders and 8 assistants, are working at a summer camp. A particular activity requires the participation of 5 employees, at least 4 of whom must be leaders.

9. Which of the following expressions can be used to determine the number of possible participant groups?

A. $_5C_4 \cdot {_8}C_1 + {_5}C_5$

B. $_5C_4 \cdot {_8}C_1 \cdot {_5}C_5$

C. $_5C_4 + {_5}C_5$

D. $_5C_4 \cdot {_5}C_5$

10. How many distinct 4-letter arrangements can be made using the letters in the word **SCRIPT**?

    A. 720

    B. 360

    C. 24

    D. 15

11. The simplified form of the expression $\dfrac{3x^2 - 3x}{x^2 - 1}$, and the restriction on x, are, respectively,

    A. $3 + 3x$ and $x \neq 1$

    B. $3 + 3x$ and $x \neq -1, 1$

    C. $\dfrac{3x}{x+1}$ and $x \neq -1$

    D. $\dfrac{3x}{x+1}$ and $x \neq -1, 1$

Use the following information to answer numerical-response question 5.

An expression equivalent to $\dfrac{3x-8}{x+3}$, $x \neq -3, 0$, is written in the form $\dfrac{Ax^B - 16x}{Cx^2 + Dx}$, where A, B, C, and D represent single-digit whole numbers.

Numerical Response

5. The value of

A is _____ (Record in the **first** column)

B is _____ (Record in the **second** column)

C is _____ (Record in the **third** column)

D is _____ (Record in the **fourth** column)

(Record your answer in the numerical-response section on the answer sheet.)

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12. The number of distinct non-permissible values in the expression
$\dfrac{(x-1)(x+7)}{(x+7)(x-9)} \div \dfrac{x}{(x-9)(x+8)}$ is

   A. 5

   B. 4

   C. 3

   D. 2

13. Which of the following expressions is equivalent to $\dfrac{1}{x+2} - \dfrac{4}{x}$, where $x \neq -2, 0$?

   A. $\dfrac{5x+8}{x(x+2)}$

   B. $\dfrac{5x+2}{x(x+2)}$

   C. $\dfrac{-3x+2}{x(x+2)}$

   D. $\dfrac{-3x-8}{x(x+2)}$

Use the following information to answer numerical-response question 6.

A rational expression in the form $\dfrac{A}{B} \cdot \dfrac{3m^3}{6n^2}$ can be simplified to $\dfrac{m}{n}$, where $m \neq 0$, $n \neq 0$. Celine knows that the original rational expression can be formed by selecting expressions for A and B from the tables below.

Possible Expression for A	Code
n^2	1
n^3	2
$2n^2$	3
$2n^3$	4

Possible Expression for B	Code
nm	5
nm^2	6
n^2m^2	7

Numerical Response

6. Record the code numbers that identify the expressions Celine can select for A and B to form the original rational expression. (There is more than one correct answer.)

Code Number: _____ _____
    Expression: A B

(Record **both digits** of your answer in the numerical-response section on the answer sheet.)

When a scuba diver dives below 33 ft, the time, t, in minutes, that she can remain underwater and then safely surface without a decompression stop can be modelled by the function

$$t = \frac{1\,700}{d - 33}$$

where d represents the maximum depth of the dive in feet.

14. During a 50-minute dive, the maximum depth, to the nearest foot, to which the diver can dive and then safely surface without a decompression stop is

   A.   20 ft
   B.   34 ft
   C.   67 ft
   D.   100 ft

While correctly solving the rational equation $\dfrac{3x + 6}{5} + \dfrac{5x}{x + 2} = 2x$ algebraically, a student wrote an equivalent quadratic equation of the form $ax^2 + bx + c = 0$.

15. The equivalent quadratic equation could have been

   A. $2x^2 + 6x - 6 = 0$
   B. $3x^2 + 35x + 12 = 0$
   C. $7x^2 - 17x - 12 = 0$
   D. $10x^2 + 12x - 6 = 0$

Four Expressions, where $R > 1$

Expression I $3\log_R R$

Expression II $\log_R(R \cdot R)$

Expression III $\log_R\left(\dfrac{R}{R^2}\right)$

Expression IV $\log_R\left(\dfrac{R}{R}\right)$

16. *The expression with the largest numeric value is numbered* ____**i**____ *, and the expression with the smallest numeric value is numbered* ____**ii**____ *.*

The statement above is completed by the information in row

Row	*i*	*ii*
A.	I	III
B.	I	IV
C.	II	III
D.	II	IV

Numerical Response

7. When the expression $\log_b 32 - \log_b 7$, $b > 1$, is written in the form $\log_b a$, the value of a, to the nearest hundredth, is _____.

(Record your answer in the numerical-response section on the answer sheet.)

17. The logarithmic equation $2 \log_a b = c$, where $a > 1$, expressed in exponential form, could be

    A. $a^c = 2b$

    B. $a^c = b^2$

    C. $a^{2b} = c$

    D. $a^b = c^2$

Use the following information to answer question 18.

Antoine incorrectly solved the equation $4^{(2-x)} = 8^{(x+1)}$. His work is shown below.

Step I	$(2^2)^{(2-x)} = (2^3)^{(x+1)}$
Step II	$2^{(4-x)} = 2^{(3x+1)}$
Step III	$4 - x = 3x + 1$
Step IV	$-4x = 3$
Step V	$x = -0.75$

18. Antoine's **first** recorded error is shown in

    A.   Step I

    B.   Step II

    C.   Step III

    D.   Step IV

Use the following information to answer question 19.

Statistics Canada has been tracking the number of farming families in Alberta. In 2005, there were 31 850 farming families. Since that time, the number of farming families has decreased at an average rate of 3.3% annually.

19. Which of the following exponential functions could be used to model the number of farming families, F, in Alberta, t years after 2005?

    A. $31\ 850 = F(1.033)^t$

    B. $31\ 850 = F(0.967)^t$

    C. $F = 31\ 850(1.033)^t$

    D. $F = 31\ 850(0.967)^t$

A design for a new sports arena is tested to measure the pressure placed on the building's exterior, in pounds of force per square foot (lb/ft^2), for various wind speeds, in miles per hour (mph). The data are listed in the table below.

Wind Speed (mph)	Pressure (lb/ft^2)
0	0
40	6.7
50	10.4
60	14.9
70	18.6
80	24.2

These data can be modelled by a quadratic regression function of the form

$$y = ax^2 + bx + c$$

where x is the wind speed, in miles per hour, and y is the pressure, in pounds per square foot.

Numerical Response

8. Based on the quadratic regression function, the pressure created by a wind speed of 20 mph, to the nearest tenth of a pound per square foot, is _____ lb/ft^2.

(Record your answer in the numerical-response section on the answer sheet.)

20. *The graph of a cubic function of the form* $y = ax^3 + bx^2 + cx + d$, *where a < 0 and d > 0, extends from Quadrant ___i___ to Quadrant ___ii___ and the sign of the y-intercept is ___iii___ .*

The statement above is completed by the information in row

Row	*i*	*ii*	*iii*
A.	2	4	positive
B.	2	4	negative
C.	3	1	positive
D.	3	1	negative

Use the following information to answer question 21.

A science class is investigating trajectories by launching balls with a catapult. The pathway of one particular ball can be modelled by the quadratic function

$$h = -4.9t^2 + 7t + 0.5$$

where h is the height of the ball above the ground, in metres, and t is the time elapsed since the launch, in seconds.

21. Which of the following rows identifies the domain and range of the function in this context?

Row	Domain	Range
A.	$0 \le t \le 0.7$	$0 \le h \le 3.0$
B.	$0 \le t \le 0.7$	$0 \le h \le 0.5$
C.	$0 \le t \le 1.5$	$0 \le h \le 3.0$
D.	$0 \le t \le 1.5$	$0 \le h \le 0.5$

A Ferris wheel at a local fair has a diameter of 68 m and the maximum height above the ground reached by a seat is 74 m.

When discussing the graph of a sinusoidal function that models the height of the seat above the ground during the ride, six students made the following statements.

Statement 1 The median value is 34 m.

Statement 2 The median value is 40 m.

Statement 3 The minimum value is 0 m.

Statement 4 The minimum value is 6 m.

Statement 5 The amplitude is 34 m.

Statement 6 The amplitude is 68 m.

22. The three statements above that are true are numbered

    A. 1, 4, and 5

    B. 1, 3, and 6

    C. 2, 4, and 6

    D. 2, 4, and 5

The regular rise and fall of the ocean's water level can limit the times at which ships can enter and exit a harbour. On a given day, the depth of the water in a particular harbour, d, in metres, can be modelled by the sinusoidal function

$$d = 2.35 \sin(0.51t + 4.11) + 13.13$$

where t is the time, in hours after midnight.

23. If a particular ship requires a minimum water depth of 13.15 m to enter the harbour on the given day, the time at which it can first enter the harbour, to the nearest hundredth of an hour after midnight, is

    A. 1.18 h

    B. 4.28 h

    C. 6.13 h

    D. 10.82 h

An electric toy car is travelling around a circular track at a constant speed. A ruler is positioned beside the track as shown in the diagram below.
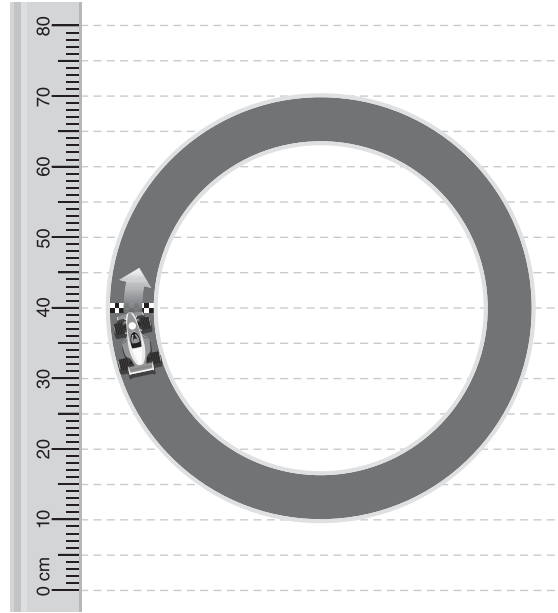
The position of the car, measured in centimetres with the ruler, can be modelled by the sinusoidal function

$$p = 30 \sin \frac{\pi}{2} t + 40$$

where p is the position of the car, in centimetres, and t is the elapsed time, in seconds.

24. *The amplitude of the sinusoidal function is* ____**i**____ *, and the time it takes for the car to travel around the track once is* ____**ii**____ *.*

The statement above is completed by the information in row

Row	*i*	*ii*
A.	30 cm	4 s
B.	30 cm	2 s
C.	60 cm	4 s
D.	60 cm	2 s

Written-response question 1 begins on the next page.

In humans, some characteristics are determined by genes, which occur in pairs. A pair of genes can consist of any combination of dominant genes, represented by upper case letters, and recessive genes, represented by lower case letters. For example, a person may have two dominant genes (*DD*), two recessive genes (*dd*), or a dominant and a recessive gene (*Dd*).

If a person has one or two dominant genes, then the person has the dominant characteristic. Free earlobes and a widow's peak hairline, illustrated below, can both be considered dominant characteristics.

Free earlobes   Attached earlobes   Widow's peak   Straight
                                              hairline   hairline

Two parents are planning to have a child. Each parent has the genes *Ee* for earlobes. The mother has the genes *Hh* and the father has the genes *hh* for hairline. The tables below show the sample spaces for the genes that the child may have for these two characteristics.

Earlobes

Mother

Father		*E*	*e*
	E	*EE*	*Ee*
	e	*Ee*	*ee*

Hairline

Mother

Father		*H*	*h*
	h	*Hh*	*hh*
	h	*Hh*	*hh*

Written Response—7 marks

1. **a.** State the odds in favour of a child of these parents having free earlobes.

b. **Determine** the probability that a child of these parents has free earlobes and a widow's peak hairline.

c. State whether the events of the child having free earlobes and the child having a widow's peak hairline are mutually exclusive or non-mutually exclusive. **Justify** your response.

Tongue rolling can also be considered a dominant characteristic. Based on the parents' genes, the probability that any one of their children will be able to roll their tongue is $\frac{3}{4}$.

d. If these parents have 2 children, **determine** the probability that both children will be able to roll their tongue or both children will not be able to roll their tongue.

Written-response question 2 begins on the next page.

After Stan finishes drinking a cup of coffee on his way to work in the morning, the amount of caffeine present in his body over time is measured and shown in the table below.

Elapsed Time (h)	Amount of Caffeine (mg)
0	150.00
1	132.00
2	116.16
3	102.22
4	89.95

These data can be modelled by an exponential function of the form

$$C = a \cdot b^t$$

where t is the elapsed time, in hours, and C is the amount of caffeine in Stan's body, in milligrams.

Written Response—7 marks

2. **a.** Write the equation of the exponential function that models the amount of caffeine in Stan's body over time. Round the value of a to the nearest whole number and the value of b to the nearest hundredth.

If Stan had decided to drink two cups of coffee on his way to work in the morning, then the equation that models the amount of caffeine in Stan's body over time can be written as a new exponential function.

b. **Compare** the equation of the new function to the equation of the original function in terms of the information about the amount of caffeine in Stan's body over time.

Use the following information to answer the next part of the question.

For Leila, caffeine has a half-life of approximately 6 h. If Leila ingested 250 mg of caffeine, the amount of caffeine in her body over time can be modelled by the exponential function

$$R = 250\left(\frac{1}{2}\right)^{\frac{t}{6}}$$

where R is the amount of caffeine in Leila's body, in milligrams, after t hours.

c. Leila no longer experiences the effects of caffeine once the amount of caffeine in her body drops to 50 mg. **Algebraically determine** the amount of time, to the nearest hour, that it would take Leila to no longer experience the effects of the caffeine ingested.

d. In a poll of 82 people in her office, Leila found that on a regular basis 63 people consumed coffee, 22 people consumed tea, and 15 people did not consume either. **Determine** the number of people in Leila's poll who consumed both coffee and tea on a regular basis.

Written-response Question 1
Sample Solution

Use the following information to answer written-response question 1.

In humans, some characteristics are determined by genes, which occur in pairs. A pair of genes can consist of any combination of dominant genes, represented by upper case letters, and recessive genes, represented by lower case letters. For example, a person may have two dominant genes (*DD*), two recessive genes (*dd*), or a dominant and a recessive gene (*Dd*).

If a person has one or two dominant genes, then the person has the dominant characteristic. Free earlobes and a widow's peak hairline, illustrated below, can both be considered dominant characteristics.

Free earlobes   Attached earlobes   Widow's peak   Straight
                                            hairline   hairline

Two parents are planning to have a child. Each parent has the genes *Ee* for earlobes. The mother has the genes *Hh* and the father has the genes *hh* for hairline. The tables below show the sample spaces for the genes that the child may have for these two characteristics.

Earlobes

Mother

	E	*e*
E	*EE*	*Ee*
e	*Ee*	*ee*

Father (rows *E*, *e*)

Hairline

Mother

	H	*h*
h	*Hh*	*hh*
h	*Hh*	*hh*

Father (rows *h*, *h*)

Written Response—7 marks

1. **a.** State the odds in favour of a child of these parents having free earlobes.

b. **Determine** the probability that a child of these parents has free earlobes and a widow's peak hairline.

A possible solution to part b

$$P(E \text{ and } H) = \frac{3}{4} \times \frac{2}{4}$$
$$= \frac{6}{16}$$

The probability that a child of these parents has free earlobes and a widow's peak hairline is $\frac{3}{8}$ or 0.375.

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c. State whether the events of the child having free earlobes and the child having a widow's peak hairline are mutually exclusive or non-mutually exclusive. **Justify** your response.

A possible solution to part c

Since the child can have both characteristics of free earlobes and a widow's peak hairline, these events are non-mutually exclusive.

OR

Justify mathematically using probability; i.e., $\frac{3}{4} + \frac{2}{4} = \frac{5}{4}$. Since the probability of the child having free earlobes or a widow's peak hairline cannot be greater than 1, this means that there are common elements that belong to both sets (i.e., a child having both characteristics of free earlobes and a widow's peak hairline) that have been counted twice. Therefore, these two events are non-mutually exclusive.

Tongue rolling can also be considered a dominant characteristic. Based on the parents' genes, the probability that any one of their children will be able to roll their tongue is $\frac{3}{4}$.

d. If these parents have 2 children, **determine** the probability that both children will be able to roll their tongue or both children will not be able to roll their tongue.

A possible solution to part d

Either both children are able to roll their tongue or both are not.

$$P(R \text{ and } R) \text{ or } P(\text{not } R \text{ and not } R) = \left(\frac{3}{4} \times \frac{3}{4}\right) + \left(\frac{1}{4} \times \frac{1}{4}\right)$$

$$= \frac{9}{16} + \frac{1}{16}$$

$$= \frac{10}{16}$$

The probability that both children are able to roll their tongue or both children are not able to roll their tongue is $\frac{5}{8}$ or 0.625.

Written-response Question 2
Sample Solution

Use the following information to answer written-response question 2.

After Stan finishes drinking a cup of coffee on his way to work in the morning, the amount of caffeine present in his body over time is measured and shown in the table below.

Elapsed Time (h)	Amount of Caffeine (mg)
0	150.00
1	132.00
2	116.16
3	102.22
4	89.95

These data can be modelled by an exponential function of the form

$$C = a \cdot b^t$$

where t is the elapsed time, in hours, and C is the amount of caffeine in Stan's body, in milligrams.

Written Response—7 marks

2. **a.** Write the equation of the exponential function that models the amount of caffeine in Stan's body over time. Round the value of a to the nearest whole number and the value of b to the nearest hundredth.

A possible solution to part a

$C = 150(0.88)^t$

Use the following additional information to answer the next part of the question.

If Stan had decided to drink two cups of coffee on his way to work in the morning, then the equation that models the amount of caffeine in Stan's body over time can be written as a new exponential function.

b. Compare the equation of the new function to the equation of the original function in terms of the information about the amount of caffeine in Stan's body over time.

A possible solution to part b

If Stan decides to drink two cups of coffee on his way to work in the morning, then the a value in the equation of the new function will double since the initial amount of caffeine in Stan's body has doubled; i.e., 150 mg will increase to 300 mg. The b value in the equation of the new function remains the same since the hourly rate of decay of caffeine in Stan's body over time (i.e., 0.88) does not change.

For Leila, caffeine has a half-life of approximately 6 h. If Leila ingested 250 mg of caffeine, the amount of caffeine in her body over time can be modelled by the exponential function

$$R = 250\left(\frac{1}{2}\right)^{\frac{t}{6}}$$

where R is the amount of caffeine in Leila's body, in milligrams, after t hours.

c. Leila no longer experiences the effects of caffeine once the amount of caffeine in her body drops to 50 mg. **Algebraically determine** the amount of time, to the nearest hour, that it would take Leila to no longer experience the effects of the caffeine ingested.

A possible solution to part c

$$R = 250\left(\frac{1}{2}\right)^{\frac{t}{6}}$$

$$50 = 250\left(\frac{1}{2}\right)^{\frac{t}{6}}$$

$$0.2 = \left(\frac{1}{2}\right)^{\frac{t}{6}}$$

$$\log_{\frac{1}{2}} 0.2 = \frac{t}{6}$$

$$6 \times \log_{\frac{1}{2}} 0.2 = t$$

$$13.9315\ldots = t$$

$$14 = t$$

∴ It will take 14 hours for Leila to no longer experience the effects of the caffeine ingested.

d. In a poll of 82 people in her office, Leila found that on a regular basis 63 people consumed coffee, 22 people consumed tea, and 15 people did not consume either. **Determine** the number of people in Leila's poll who consumed both coffee and tea on a regular basis.
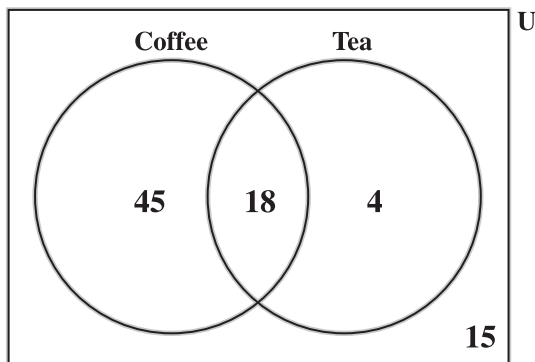

A possible solution to part d

$82 - 15 = 67$ people consumed either coffee or tea

$63 + 22 = 85$ people consumed either coffee or tea, with the number of people who consumed both coffee and tea counted twice

$\therefore 85 - 67 = 18$ people consumed both coffee and tea

OR

18 people consumed both coffee and tea

OR

Let x represent the number of people who consumed both coffee and tea on a regular basis.

$(63 - x) + x + (22 - x) + 15 = 82$

$$100 - x = 82$$

$$18 = x$$

Scoring Guide for Written-response Question 1

WR 1 Part a

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	In the response, the student: • incorrectly states the odds in favour OR • states a probability.
0.5		
1	In the response, the student applies appropriate mathematical knowledge to find a complete and correct solution.	In the response, the student: • correctly states the odds in favour.

Note: Any equivalent odds statement can receive full marks.

WR 1 Part b

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	In the response, the student: • determines an incorrect probability OR • writes either $\frac{3}{4}$ only or $\frac{2}{4}$ only with no probability statement.
0.5		*For example, the student could:* • *state either the correct probability of a child having free earlobes only or the correct probability of a child having a widow's peak only* *OR* • *state the correct answer with no supporting work.*
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student: • determines the probability of a child having free earlobes and the probability of a child having a widow's peak, but: 　– does not calculate the probability of having both 　or 　– incorrectly calculates the probability of having both OR • determines the correct probability of only one of the events, and continues with a correct process using both the correct and incorrect probabilities to provide a solution.
1.5		*For example, the student could:* • *correctly determine the probability of a child having both characteristics, but reduce incorrectly.*
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • correctly determines the probability of a child having both free earlobes and a widow's peak.

Notes:
• Full marks can be awarded even if the student does not reduce probability to lowest terms.
• Showing a sample space is an appropriate strategy.

WR 1 Part c

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	In the response, the student • states that the events are mutually exclusive OR • provides an incorrect or contradictory justification.
0.5		*For example, the student could:* • *correctly state that the events are non-mutually exclusive only* *OR* • *correctly state that the events are non-mutually exclusive and provide an irrelevant and non-contradictory justification.*
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student • correctly states that the events are non-mutually exclusive and provides: – an incomplete justification or – the definition of non-mutually exclusive events.
1.5		*For example, the student could:* • *correctly state that the events are non-mutually exclusive and provide a justification that lacks clarity.*
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • correctly states that the events are non-mutually exclusive and provides clear and correct justification, making reference to the context.

WR 1 Part d

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	In the response, the student • states an incorrect probability.
0.5		*For example, the student could:* • *correctly determine the probability of one case only* *OR* • *identify the complement and recognize cases, but determine the probabilities incorrectly* *OR* • *state the correct answer with no supporting work.*
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student • correctly determines the probability of each case, but does not combine them or combines them incorrectly.
1.5		*For example, the student could:* • *correctly determine the probability of both cases, but reduce incorrectly.*
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • correctly determines the probability that both children will be able to roll their tongue or both children will not be able to roll their tongue.

Notes:
• A tree diagram is an appropriate strategy. A complete and correct tree diagram will be awarded at least 1.
• Full marks can be awarded even if the student does not reduce the final probability to lowest terms.

Scoring Guide for Written-response Question 2

WR 2 Part a

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	In the response, the student: • writes an incorrect function.
0.5		*For example, the student could:* • *correctly state the parameter values, but does not write a function.*
1	In the response, the student applies appropriate mathematical knowledge to find a complete and correct solution.	In the response, the student: • writes the correct equation of the exponential function.

Note: Rounding is a minor error and a student can still receive full marks.

WR 2 Part b

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	In the response, the student • provides an incorrect comparison OR • provides the definitions of both parameter values only.
0.5		*For example, the student could:* • *provide a correct comparison of one of the parameter values in the exponential functions, but make no reference to the context.*
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student • provides a correct comparison of one of the parameter values in the exponential functions, making reference to the context OR • writes the correct equation of the new exponential function only OR • provides a correct comparison of both parameter values in the exponential functions, but makes no reference to the context.
1.5		*For example, the student could:* • *provide a correct comparison of both parameter values in the exponential functions, but make reference to the context for only one of the parameter values* *OR* • *write the correct equation of the new exponential function and provide a correct comparison of one of the parameter values, making reference to the context.*
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • provides a complete and correct comparison of both parameter values in the exponential functions, making reference to the context.

Note: Students do not need to write the new exponential function in order to receive full marks.

WR 2 Part c

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	In the response, the student • substitutes 50 in for t OR • states the correct answer with no supporting work.
0.5		*For example, the student could:* • *correctly substitute 50 in for R and solve graphically or divide both sides by 250* *OR* • *solve correctly, using reasoning.*
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student • correctly substitutes 50 in for R, but makes an error such as – incorrectly converting to logarithmic form (e.g., $\log_{0.2}\frac{1}{2}$) or – taking the log of both sides of the equation, but incorrectly isolating t OR • incorrectly multiplies 250 by $\frac{1}{2}$, but continues on using a correct algebraic process to get the answer 4.86 hrs (5 hrs)
1.5		*For example, the student could:* • *correctly determine the number of hours algebraically, but round the final answer incorrectly* *OR* • *correctly show all algebraic work, but obtain an incorrect answer.*
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • correctly determines the number of hours algebraically.

WR 2 Part d

Score	General Description	Specific Description
NR	No response is provided.	
0	In the response, the student does not address the question or provides a solution that is invalid.	In the response, the student • draws an incorrect Venn diagram.
0.5		*For example, the student could:* • *start a Venn diagram and label at least the 15 correct* *OR* • *correctly state the n(coffee or tea)* *OR* • *state the correct answer with no supporting work.*
1	In the response, the student demonstrates basic mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a partial solution.	In the response, the student • correctly completes two regions in a Venn diagram OR • provides at least two relevant and accurate calculations.
1.5		*For example, the student could:* • *correctly determine that 18 people consume both, but with supporting work that lacks clarity* *OR* • *correctly draw a Venn diagram, but without clearly indicating the answer.*
2	In the response, the student demonstrates complete mathematical understanding of the problem by applying an appropriate strategy or relevant mathematical knowledge to find a complete and correct solution.	In the response, the student • correctly determines that 18 people consume both coffee and tea, clearly indicating the final answer.

Examples of the Standards for Students' Work

This section provides sample student responses and scoring rationales as they relate to the general scoring guide. These examples are intended to inform teachers and students of how the scoring guide is applied to specific questions and to provide examples of Mathematics 30–2 work that meet or exceed the acceptable standard for student achievement. Teachers and students should note that directing words are bolded in written-response questions on diploma examinations. A list of these directing words and definitions can be found in the *Mathematics 30–2 Information Bulletin*.

Sample response 1

Use the following information to answer written-response question 1.

In humans, some characteristics are determined by genes, which occur in pairs. A pair of genes can consist of any combination of dominant genes, represented by upper case letters, and recessive genes, represented by lower case letters. For example, a person may have two dominant genes (*DD*), two recessive genes (*dd*), or a dominant and a recessive gene (*Dd*).

If a person has one or two dominant genes, then the person has the dominant characteristic. Free earlobes and a widow's peak hairline, illustrated below, can both be considered dominant characteristics.

| Free earlobes | Attached earlobes | Widow's peak hairline | Straight hairline |

Two parents are planning to have a child. Each parent has the genes *Ee* for earlobes. The mother has the genes *Hh* and the father has the genes *hh* for hairline. The tables below show the sample spaces for the genes that the child may have for these two characteristics.

Earlobes

		Mother	
		E	*e*
Father	*E*	*EE*	*Ee*
	e	*Ee*	*ee*

Hairline

		Mother	
		H	*h*
Father	*h*	*Hh*	*hh*
	h	*Hh*	*hh*

Written Response—7 marks

1. a. State the odds in <u>favour</u> of a child of these parents having free earlobes.

EE, Ee, Ee

$\dfrac{3}{4}$ desired total

3 : 1

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b. **Determine** the probability that a child of these parents has free earlobes and a widow's (X)
peak hairline.

$$\frac{3}{4} \times \frac{1}{2} \qquad \frac{3}{8}$$

37.5% chance
child has free earlobes
+ widow's peak

X

c. State whether the events of the child having free earlobes and the child having a widow's
peak hairline are mutually exclusive or non-mutually exclusive. **Justify** your response.

Non mutually exclusive, because
a child could have the dominant earlobe
gene AND the widow's peak gene,

there is a 37.5% chance this
could happen.

Use the following information to answer the next part of the question.

> Tongue rolling can also be considered a dominant characteristic. Based on the parents' genes,
> the probability that any one of their children will be able to roll their tongue is $\frac{3}{4}$.

d. If these parents have 2 children, **determine** the probability that both children will be able to roll their tongue or both children will not be able to roll their tongue.

(+)

$$\frac{3}{4} \times \frac{3}{4} \qquad + \qquad \frac{1}{4} \times \frac{1}{4}$$

$$\frac{9}{16} \qquad\qquad \frac{2}{16} \quad \frac{1}{8} \quad \boxed{= 68\%}$$

$$56\% \qquad\qquad\qquad 12.5\%$$

will   won't

56% they both <u>will</u> be able to
12.5% they both <u>won't</u> be able to

Together is 68%

Total Score - 6.5 marks	Rationale
Part a: 1 mark Part b: 2 marks Part c: 2 marks Part d: 1.5 marks	The student correctly answered parts a, b and c. In part d, an error was made in calculating the probability of both children not being able to roll their tongue, but the student continued with a correct process in combining the two cases.

Sample response 2

In humans, some characteristics are determined by genes, which occur in pairs. A pair of genes can consist of any combination of dominant genes, represented by upper case letters, and recessive genes, represented by lower case letters. For example, a person may have two dominant genes (*DD*), two recessive genes (*dd*), or a dominant and a recessive gene (*Dd*).

If a person has one or two dominant genes, then the person has the dominant characteristic. Free earlobes and a widow's peak hairline, illustrated below, can both be considered dominant characteristics.

| Free earlobes | Attached earlobes | Widow's peak hairline | Straight hairline |

Two parents are planning to have a child. Each parent has the genes *Ee* for earlobes. The mother has the genes *Hh* and the father has the genes *hh* for hairline. The tables below show the sample spaces for the genes that the child may have for these two characteristics.

Earlobes

		Mother	Mother
		E	*e*
Father	*E*	*EE*	*Ee*
	e	*Ee*	*ee*

Hairline

		Mother	Mother
		H	*h*
Father	*h*	*Hh*	*hh*
	h	*Hh*	*hh*

Written Response—7 marks

1. a. State the odds in favour of a child of these parents having free earlobes.

$$3 : 1$$

b. Determine the probability that a child of these parents has free earlobes and a widow's peak hairline.

$$\frac{3}{4} \times \frac{2}{4} = .375.$$

c. State whether the events of the child having free earlobes and the child having a widow's peak hairline are mutually exclusive or non-mutually exclusive. **Justify** your response.

non mutually exclusive.
The child does not need to
have one (widows peak) to have
the other (ear lobes).

Use the following information to answer the next part of the question.

Tongue rolling can also be considered a dominant characteristic. Based on the parents' genes, the probability that any one of their children will be able to roll their tongue is $\frac{3}{4}$.

d. If these parents have 2 children, **determine** the probability that both children will be able to roll their tongue or both children will not be able to roll their tongue.

$$\frac{3}{4} \times \frac{3}{4} = .5625 \text{ probability}$$
they can roll tongue

$.4375$ probability they can't.

Total Score - 4 marks	Rationale
Part a: 1 mark **Part b: 2 marks** **Part c: 0.5 marks** **Part d: 0.5 marks**	The student correctly answered parts a and b. In part c, the correct position was taken, but the student did not provide a reasonable justification by clearly communicating their understanding of non-mutually exclusive events in this context. In part d, the student correctly determined the probability of one case only.

Sample response 3

Use the following information to answer written-response question 2.

After Stan finishes drinking a cup of coffee on his way to work in the morning, the amount of caffeine present in his body over time is measured and shown in the table below.

Elapsed Time (h)	Amount of Caffeine (mg)
0	150.00
1	132.00
2	116.16
3	102.22
4	89.95

These data can be modelled by an exponential function of the form

$$C = a \cdot b^t$$

where t is the elapsed time, in hours, and C is the amount of caffeine in Stan's body, in milligrams.

Written Response—7 marks

2. **a.** Write the equation of the exponential function that models the amount of caffeine in Stan's body over time. Round the value of a to the nearest whole number and the value of b to the nearest hundredth.

$$C = 150 \cdot 0.88^t$$

If Stan had decided to drink <u>two cups</u> of coffee on his way to work in the morning, then the equation that models the amount of caffeine in Stan's body over time can be written as a new exponential function.

b. Compare the equation of the new function to the equation of the original function in terms of the information about the amount of caffeine in Stan's body over time.

Time	Caffic
0	300
1	264
2	232.2
3	204.44
4	179.9

$$C = 300 \cdot 0.88^t$$

For Leila, caffeine has a half-life of approximately 6 h. If Leila ingested 250 mg of caffeine, the amount of caffeine in her body over time can be modelled by the exponential function

$$R = 250\left(\frac{1}{2}\right)^{\frac{t}{6}}$$

where R is the amount of caffeine in Leila's body, in milligrams, after t hours.

c. Leila no longer experiences the effects of caffeine once the amount of caffeine in her body drops to 50 mg. **Algebraically determine** the amount of time, to the nearest hour, that it would take Leila to no longer experience the effects of the caffeine ingested.

$$50 = 250\left(\frac{1}{2}\right)^{\frac{t}{6}}$$
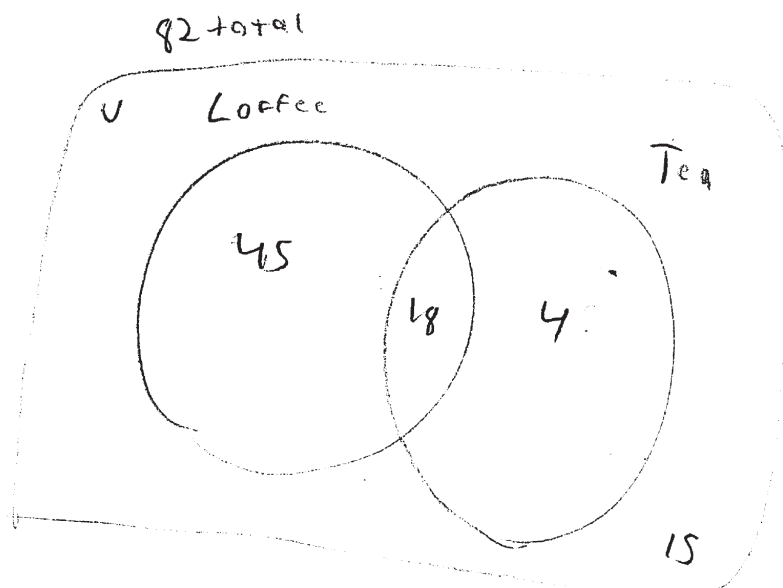
$$50 = 125^{\frac{t}{6}}$$

$$\log_{125}^{50} = \frac{t}{6}$$

$$6 \cdot 0.81 = \frac{t}{6}$$

$$4.86 = t$$

$$\boxed{5\text{ hours} = t}$$

d. In a poll of 82 people in her office, Leila found that on a regular basis 63 people consumed coffee, 22 people consumed tea, and 15 people did not consume either. **Determine** the number of people in Leila's poll who consumed both coffee and tea on a regular basis.

3

92 total

U   Coffee

Tea

45

18   4

15

~~63+15-92~~

92 - (63+15) = 4

$63 + 22 + 15 = 100$
$ -82$
$\overline{18}$

$63 - 18 = 45$

$22 - 18 = 4$

$C \cup T = 67$

67 People drink both coffee and tea.

Total Score - 4.5 marks	Rationale
Part a: 1 mark Part b: 1 mark Part c: 1 mark Part d: 1.5 marks	The student correctly answered part a. In part b, the student's response reflects a basic understanding of the problem as only the new equation of the function was written and a complete comparison, referencing the context, was not provided. In part c, the student starts with the error of multiplying the coefficient and base of the power, but continues with an otherwise correct algebraic process. In part d, the student shows a correct process, but states an incorrect answer.